EC 3210 Solutions

Assignment 10

11.1. A HeNe laser has a 1500 MHz linewidth. Calculate the length of the resonator that is required to ensure that only one mode will oscillate.

We are given a HeNe laser ($\lambda = 632.8 \times 10^{-9}$) with $\Delta\nu = 1.5 \times 10^{9}$. We want to find L such that

$$\frac{c}{2L} > \Delta\nu \tag{1}$$

$$\frac{1}{L} > \frac{2\Delta\nu}{c}$$

$$L < \frac{c}{2\Delta\nu} = \frac{3.0 \times 10^{8}}{(2)(1.5 \times 10^{9})} < 9.99 \times 10^{-2} \text{ m} = 9.99 \text{ cm}\,.$$

11.3.

a. Calculate the pulse width produced by a mode-locked Nd laser with a 25 cm cavity and 300 modes.

b. Estimate the linewidth of the source.

c. Calculate the PRF of the pulse train.

a. Mode-locked Nd laser with $L = 25 \times 10^{-2}$ and 300 modes ($N = 300$).

$$\Delta t = \frac{1}{\Delta\nu} = \frac{1}{\frac{Nc}{2nL}} = \frac{1}{\frac{(300)(3.0\times10^{8})}{(2)(1)(25\times10^{-2})}} = 5.56 \times 10^{-12} = 5.56 \text{ ps}\,. \tag{2a}$$

b. The linewidth $\Delta\nu$ is

$$\Delta\nu = \frac{Nc}{2nL} = \frac{(300)(3.0 \times 10^{8})}{(2)(1)(25 \times 10^{-2})} = 1.8 \times 10^{11} = 180 \text{ GHz}\,. \tag{2b}$$

c. The PRF is

$$\text{PRF} = \frac{1}{T} = \frac{c}{2nL} = 6 \times 10^{8} \text{ pps}\,. \tag{2c}$$

11.5. A Q-switched ruby laser (λ694.3 nm) is to be used to measure distances to the moon (approximately 3.8×10^{8} m). The peak power of the laser pulse is 10 megawatts. The 50 ns pulse will be retroreflected by a set of corner cubes left there by astronauts. We need to perform a power analysis to see if this is feasible.

a. An astronomical telescope (diameter of 300 cm) will be used as beam collimator to reduce the beam divergence of the laser. Calculate the approximate beam diameter on the moon if beam diameter at the laser is 1 cm with a full angle beam divergence of 0.1 mrad.

b. The square retroreflector array is 50 cm on a side. Calculate the fraction of the laser power that will be intercepted and reflected by the array.

c. The beam divergence of the reflected light is determined by the size of the reflector elements in the array. Each element has a circular shape with a diameter of 4.0 cm. Calculate the fraction of the original power from the laser that will be intercepted and received by the telescope on earth.

d. If our most sensitive detector can detect 1×10^{-12} W, is it feasible to use this laser for the measurement?

e. How long will it take a pulse to traverse the round-trip distance? If the measurement uncertainty is twice the distance represented by a pulse, calculate the measurement uncertainty for this laser.

We have a Q-switched ruby laser ($\lambda = 694 \times 10^{-9}$) to be used to go from the earth to the moon ($R = 3.8 \times 10^8$ m). The pulse width is 50×10^{-9} s and the peak power is 10 MW.

a. The beam entering the telescope has $d_{\text{in}} = 2w_{\text{in}} = 1 \times 10^{-2}$ and a beam divergence of 0.1 mr. The output beam will have to pass through an aperture of 3 m diameter. We want to fill up the aperture as much as possible to minimize the output beam divergence. Since we know that $D \le 3w$ for a Gaussian beam, we will want to use the equality.

$$\begin{aligned} D &= 3w \\ 3 &= 3w \\ w &= 1.00 \text{ m}\,. \end{aligned} \tag{3}$$

The output beam divergence is found from

$$\frac{\phi_{\text{out}}}{\phi_{\text{in}}} = \frac{w_{\text{in}}}{w_{\text{out}}}\,, \tag{4a}$$

so

$$\phi_{\text{out}} = \frac{w_{\text{in}}\phi_{\text{ in}}}{w_{\text{out}}} = \frac{(0.5)(1 \times 10^{-4})}{100} = 0.5 \times 10^{-6} = 0.5\ \mu\text{r}\,. \tag{4b}$$

We need to check if the moon is in the far-field of the collimator output.

$$z_R = \frac{\pi w_0^2}{\lambda} = \frac{\pi(100 \times 10^{-2})^2}{694 \times 10^{-9}} = 4.5 \times 10^6 \text{ m}\,. \tag{5}$$

At 3.8×10^8 m, the moon *is* in the far-field. Since we have calculated the full-angle beam divergence, the diameter of the spot on the moon d_{moon} is found from

$$d_{\text{moon}} = R\,\phi_{\text{out}} = (3.8 \times 10^8)(0.5 \times 10^{-6}) = 1.9 \times 10^2 = 190.0 \text{ m}\,. \tag{6}$$

b. The fraction of the power intercepted by the retroreflector is

$$\%_{\text{intercepted}} = 100 \times \frac{A_{\text{retroreflector}}}{A_{\text{beam}}} = 100 \times \frac{(50 \times 10^{-2})^2}{\pi\left(\frac{(190)^2}{4}\right)} = 8.82 \times 10^{-4}\ \%\,. \tag{7}$$

Only a very small fraction is intercepted.

c. The reflective elements have a diameter of 4 cm. Assuming that the wave is a plane wave by the time that it reaches the moon, the beam divergence of the retroreflected wave will be

$$\phi_{\text{retroreflector}} = \frac{2.44\lambda}{d} = \frac{2.44(694 \times 10^{-9})}{4 \times 10^{-2}} = 4.23 \times 10^{-5} \text{ radians}\,. \tag{8}$$

The diameter of the beam when it returns to earth is

$$d_{\text{earth}} = \phi_{\text{retro}} R = (4.23 \times 10^{-5})(3.8 \times 10^{8}) = 16.07 \times 10^{3} = 16.07 \text{ km}\,. \tag{9}$$

The fraction intercepted by the telescope will be

$$\begin{aligned} \%_{\text{intercepted}} &= 100 \times \frac{A_{\text{telescope}}}{A_{\text{beam}}} = 100 \times \frac{\pi\left(\frac{d_T^2}{4}\right)}{\pi\left(\frac{d_B^2}{4}\right)} \qquad (10)\\ &= \frac{d_t^2}{d_B^2} = 100 \times \left(\frac{300 \times 10^{-2}}{1.607 \times 10^{4}}\right)^2 = 3.48 \times 10^{-6}\ \%\,. \end{aligned}$$

The total fraction received is

$$\begin{aligned} \frac{P_R}{P_{\text{out}}} &= \text{fraction retroreflected} \times \text{ fraction on telescope} \qquad (11)\\ &= (8.817 \times 10^{-6})(3.48 \times 10^{-8}) = 2.85 \times 10^{-13}\,. \end{aligned}$$

d. The peak power received is

$$\begin{aligned} P_{\text{pk R}} &= \text{fraction received} \times P_{\text{pk transmitted}} \qquad (12)\\ &= (2.85 \times 10^{-13})(10 \times 10^{6}) = 2.85 \times 10^{-6} = 2.85\ \mu\text{W}\,. \end{aligned}$$

Since we can detect signals that are greater than 10^{-12} W, we should be able to *easily* detect this returned signal.

e. The time T to traverse the round-trip distance is

$$T = \frac{2R}{c} = \frac{(2)(3.8 \times 10^{8})}{3.0 \times 10^{8}} = 2.53 \text{ s}\,. \tag{13a}$$

The measurement uncertainty is twice the pulse width (in distance), so

$$\Delta L = 2c\,\Delta t = (2)(3.0 \times 10^{8})(50 \times 10^{-9}) = 30.0 \text{ m}\,. \tag{13b}$$