EC 3210 Solutions

Assignment 3

2.7 A Fabry-Perot interferometer has a spacing of 1.500 cm.

a. Calculate the free spectral range (i.e., $\Delta\sigma$).

b. Can this interferometer be used as scanning interferometer to measure the gain curve of a HeNe laser with a linewidth of 0.015 nm? Prove your answer.

Solution:

a. The free spectral range is given by

$$\Delta\sigma = \frac{c}{2d} = \frac{3.0 \times 10^8}{2(1.500 \times 10^{-2})} = 1.000 \times 10^{10} \text{ Hz} = 10.00 \text{ GHz}\,. \tag{1}$$

b. We want to use the interferometer to measure a HeNe laser line with a linewidth of 0.015 nm. For $\lambda = 632.8 \times 10^{-9}$ this is a $\Delta\nu$ of

$$\Delta\nu = \frac{c\,\Delta\lambda}{\lambda^2} = \frac{(3.0 \times 10^8)(0.015 \times 10^{-9})}{(632.8 \times 10^{-9})^2)} = 1.124 \times 10^{10} \text{ Hz} = 11.24 \text{ GHz}\,. \tag{2}$$

So we have a linewidth of 11.24 GHz that we are trying to measure with a 10 GHz wide interferometer. An ambiguity will exist in the frequency resolution as seen in Fig. 1.

Since $\Delta\sigma < \Delta\nu$, we *cannot* use this FP interferometer. We need to have $\Delta\sigma > \Delta\nu$, so d would have to be decreased (slightly).

2.8. A gas discharge emits light with a center frequency of 1×10^{14} Hz and a linewidth $\Delta\nu$ of 1 GHz. We want to design a scanning Fabry-Perot (FP) interferometer so that we are sure of measuring the frequency spectrum of this light with 100 equally-spaced data points in the frequency range extending from $\nu_0 - \frac{\Delta\nu}{2}$ to $\nu_0 + \frac{\Delta\nu}{2}$.

a. Calculate the maximum value of the mirror spacing d for this scanning FP interferometer.

b. Using the value calculated in Part a, calculate the required translation value Δd of the interferometer.

Solution: a) We want to design a scanning Fabry-Perot interferometer and ensure that $\Delta\sigma \geq \Delta\nu$, so

$$\Delta\sigma \geq \Delta\nu \tag{3a}$$

$$d \leq \frac{c}{2\Delta\nu} \leq \frac{3.0 \times 10^8}{2(1 \times 10^9)} = 1.5 \times 10^{-1} \text{ m} = 15 \text{ cm}\,. \tag{3b}$$

If we choose, arbitrarily, to let $\Delta\sigma = 4\Delta\nu$, then we would have

$$d = \frac{15}{4} = 3.75 \text{ cm}\,. \tag{4}$$

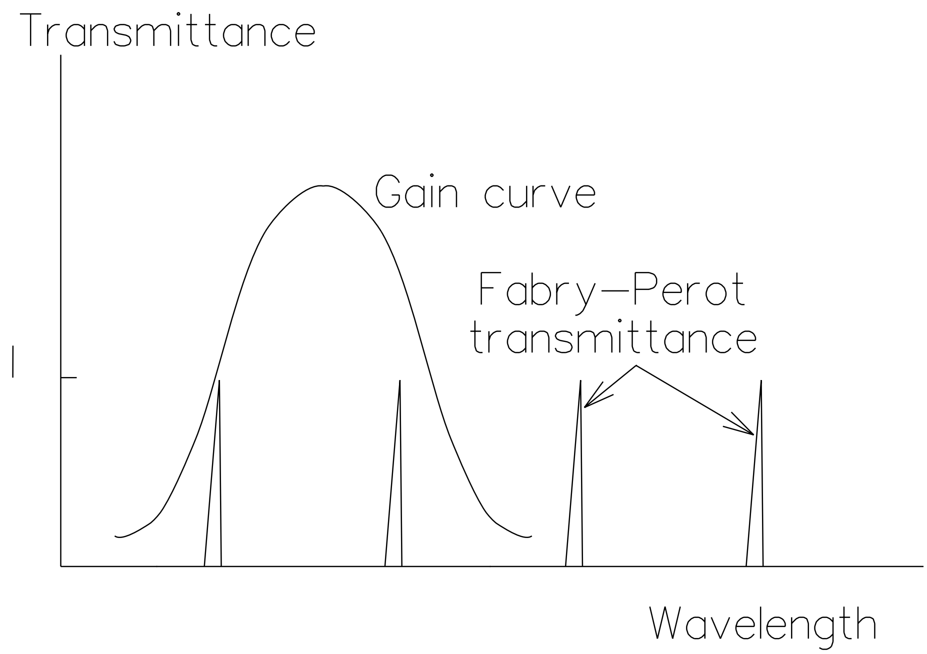

Figure 1: Problem 2.6c. Transmittance of Fabry-Perot inteferometer. Note ambiguity resulting from two transmittance peaks being located under the gain curve.

b. We want to ensure that there are 100 separate data points between the $\nu_0 + \frac{\Delta\nu}{2}$ location and the $\nu_0 + \frac{\Delta\nu}{2}$ location. The data points, then, are $\frac{\Delta\nu}{100} = \frac{1\times 10^9}{100} = 1 \times 10^7$ Hz apart, so

$$|\nu'_m - \nu_m| = \left|\nu_0 \frac{\Delta d}{d}\right| \tag{5a}$$

$$1 \times 10^7 = \nu_0 \frac{\Delta d}{d} \tag{5b}$$

$$\Delta d = \frac{d(1\times 10^7)}{\nu_0} = \frac{(3.75\times 10^{-2})(1\times 10^7)}{1\times 10^{14}} = 3.75\times 10^{-9}\ \mathrm{m} = 3.75\ \mathrm{nm} \qquad (\text{for } \Delta\sigma = 4\Delta\nu)\,. \tag{5c}$$

3.1. Consider a wave with $\tilde{E}_{\mathrm{x\ max}} = \tilde{E}_{\mathrm{y\ max}} = E_0$ and $\Delta\phi = 30°$. What kind of polarization is this?

We are given that $\tilde{E}_{\mathrm{x\ max}} = \tilde{E}_{\mathrm{y\ max}} = E_0$ and $\Delta\phi = 30°$. This gives

$$\tilde{E}_x = E_0 \cos(\omega t - kz + (\pi/6)) \tag{6a}$$

$$\tilde{E}_y = E_0 \cos(\omega t - kz) \tag{6b}$$

A plot of the E_x and E_y components is shown in Fig. 2 (with $\omega t = 0$). From this figure we can see that the polarization is elliptical.

3.2. Consider unpolarized light that passes through five linear polarizers. The sequence of polarizers and the alignment of their polarization axis from the vertical are indicated in the table below. Calculate the fraction of the incident power that is transmitted through the combination of polarizers.

Polarizer number	Angle of axis from vertical
1	0°
2	20°
3	15°
4	40°
5	120°

We find that

$$\begin{aligned}\frac{P_{\text{out }5}}{P_{\text{in }1}} &= \left(\frac{P_{\text{out }5}}{P_{\text{in }5}}\right)\left(\frac{P_{\text{out }4}}{P_{\text{in }4}}\right)\left(\frac{P_{\text{out }3}}{P_{\text{in }3}}\right)\left(\frac{P_{\text{out }2}}{P_{\text{in }2}}\right)\left(\frac{P_{\text{out }1}}{P_{\text{in }1}}\right) \qquad (7)\\ &= (\cos^2 80°)(\cos^2 25°)(\cos^2 5°)(\cos^2 20°)\left(\frac{1}{3}\right)\\ &= ((0.1737)(0.906)(0.996)(0.940))^2\,(0.333) = 7.24\times 10^{-3} = 0.724\%\,.\end{aligned}$$

We have used the fact that $P_{\text{out }1} = P_{\text{in }2}$, $P_{\text{out }2} = P_{\text{in }3}$, etc.

3.3. A vertical polarizer LP$_1$ is placed in an unpolarized laser beam that has a power P_{inc}. A second polarizer LP$_2$ is placed in the beam path with its polarization axis in the horizontal direction.

a. Calculate the fraction of P_{inc} that is transmitted through the LP$_1$ and LP$_2$ combination.

b. A third linear polarizer LP$_3$ is inserted into the beam path *in between* the first two polarizers. Its polarization axis is at a 45° angle from the vertical. Calculate the fraction of P_{inc} that is transmitted through the LP$_1$, LP$_3$, and LP$_2$ combination.

c. Let the polarization axis of the polarizer added in the previous part of the problem make an angle of θ from the vertical. (i.e., $\theta = 0°$ is a vertical orientation, and $\theta = 90°$ is a horizontal orientation. Find

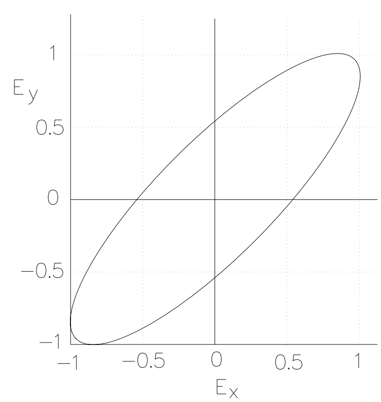

Figure 2: Locus traced by E-field of Problem 3.1.

an expression for the fraction of P_{inc} that is transmitted through the LP_1, LP_3, and LP_2 combination. (This result should be a function of θ.)

d. Plot the result of the previous part.

We have a linear polarizer LP_1 aligned vertically, followed by a second linear polarizer LP_2, aligned horizontally as shown in Fig. 3.

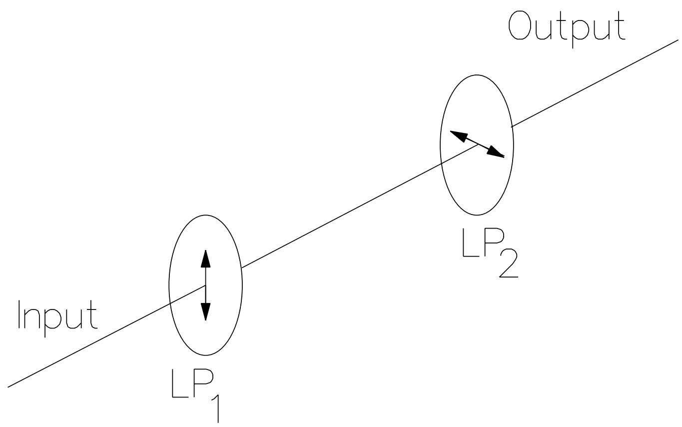

Figure 3: Polarizer geometry for Problem 3.3.

a. The output power is

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \cos^2\theta = \cos^2 90^\circ = 0\,. \tag{8}$$

No light is transmitted through the combination (as we expected).

b. We now place a third polarizer in between the first two with its axis at 45° from the vertical (see Fig. 4). We find that

$$\frac{P_1}{P_{\text{inc}}} = \frac{1}{3} \tag{9a}$$

$$\frac{P_2}{P_1} = \cos^2\theta = \cos^2 45^\circ = \frac{1}{2} \tag{9b}$$

$$\frac{P_{\text{out}}}{P_2} = \cos^2\theta' = \cos^2 45^\circ = \frac{1}{2}\,. \tag{9c}$$

So,

$$\frac{P_{\text{out}}}{P_{\text{inc}}} = \frac{P_{\text{out}}}{P_2}\frac{P_2}{P_1}\frac{P_1}{P_{\text{inc}}} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{1}{12} = 0.085\,.$$

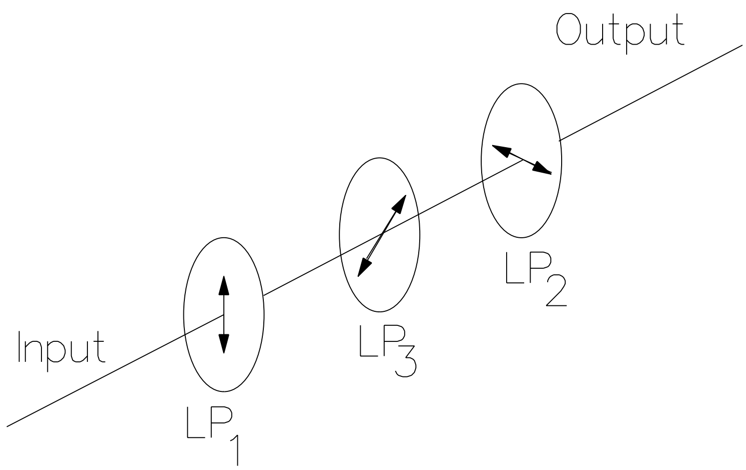

Figure 4: Polarizer geometry for Problem 3.3b.

c. For an arbitrary alignment angle of the second polarizer with an angle θ from the vertical, we have

$$\frac{P_1}{P_{\text{inc}}} = \frac{1}{3}\,, \tag{10a}$$

$$\frac{P_2}{P_1} = \cos^2\theta\,, \tag{10b}$$

$$\frac{P_{\text{out}}}{P_2} = \cos^2(90^\circ - \theta) = \sin^2\theta\,. \tag{10c}$$

So,

$$\frac{P_{\text{out}}}{P_{\text{inc}}} = \left(\frac{P_{\text{out}}}{P_2}\right)\left(\frac{P_2}{P_1}\right)\left(\frac{P_1}{P_{\text{inc}}}\right) = \cos^2(\theta)\sin^2(\theta)\left(\frac{1}{3}\right) = \frac{\cos^2(\theta)\sin^2(\theta)}{3}\,. \tag{11}$$

d. The results of Part c plot is shown in Fig. 5.

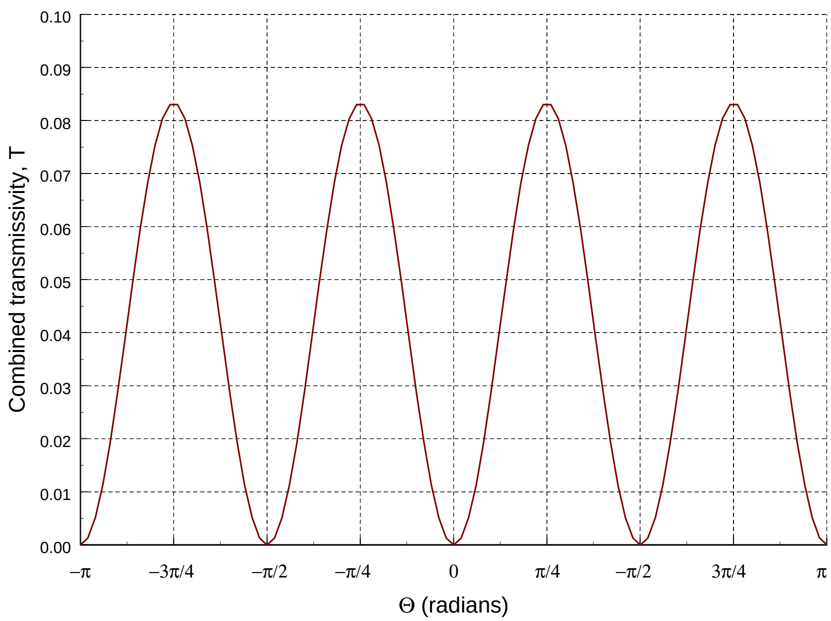

Figure 5: Problem 3.3d