

# Optical Receivers

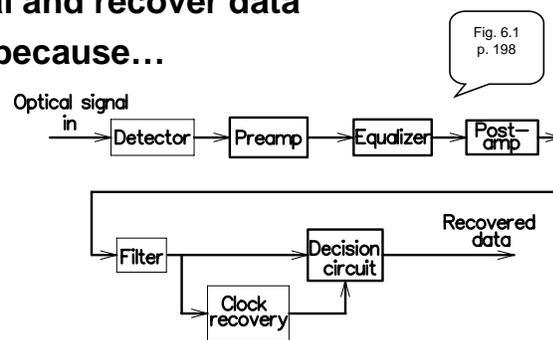
## Optical Receivers

- **Optical receiver components**

- **Optical detector**: convert modulated light into electronic signal
- **Preamp**: amplify weak electrical signal
- **Equalizer**: recover bandwidth lost in preamp
- **Postamplifier**: further amplifies signal
- **Filter**: remove unwanted frequency components
- **Clock recovery**: recover clock sent on optical signal
- **Decision circuit**: sample signal and recover data

- Receiver design is complicated because...

- Weak optical signal
- Electronic noise present



## Noise

- **Noise**
  - **Produces errors in data**
  - **Introduced by...**
    - » **Transmitter**
    - » **Channel**
      - **Fiber: zero channel noise**
    - » **Detector**
    - » **Electronic processing**
  - **Optical-detector noise different than radio and electronic detectors...**
    - » **Signal-dependent**

## Receivers: Preview

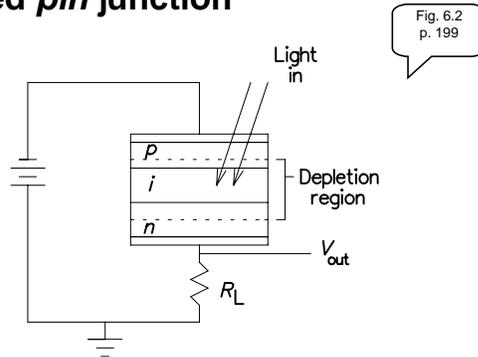
- **Detectors**
  - **Physical operating principles of...**
    - » **Pin photodiode**
    - » **Avalanche photodiode (APD)**
  - **Operating parameters**
  - **Noise performance**
  - **Operating speed**
- **Digital receiver design in terms of...**
  - **Amplifier noise**
  - **Optimization of signal-to-noise ratio**
  - **Requirements for equalization amplifier**
- **Sensitivity of optical receiver is function of...**
  - **Detector noise and...**
  - **Preamplifier choice**

## Receivers: Detector

- Converts optical input power to current output
- Photodetector properties
  - Efficiency
  - Noise
  - Spectral response
  - Speed
  - Linearity
  - IC-compatibility
  - Reliability
  - Price
- Semiconductor photodiodes
  - *pin* photodiode
  - Avalanche photodiode (APD)

## Photodiodes: Physical Principles

- Reverse-biased *pin* junction



- **Depletion region** (no free carriers) around junction
- Portion of light absorbed *in depletion region*
- Hole-electron pair created
- Pair separated and swept out by electric field
- Sensed by outside circuitry
- Number of hole-electron pairs per second freed
  - Linearly dependent on optical power
  - Electric current proportional to optical power

## Photodiodes: Spectral Response

- **Bandgap energy:** energy required to free hole-electron pair

Material	$E_g$ (eV)	$\lambda_{\max}$ ( $\mu\text{m}$ )
Si	1.14	1.09
Ge	0.67	1.85
GaAs	1.43	0.867
$\text{In}_{0.14}\text{Ga}_{0.86}\text{As}$	1.15	1.08
$\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$	0.75	1.65

- **Long- $\lambda$  response:** no sensitivity at long  $\lambda$

– **Photon energy:**  $hc/\lambda \geq E_g$

– **Long wavelength cutoff**

$$\lambda_{\max} = hc/E_g \Rightarrow 1.24/E_g' [\text{eV}]$$

» **Si:**  $\lambda_{\max} = 1.09 \mu\text{m}$ , short- $\lambda$  (visible, 800-900 nm) detector

» **InGaAs & Ge:** long- $\lambda$  (1300, 1550 nm) detectors

## Photodiodes: Spectral Response (cont.)

### • Short- $\lambda$ response

– No sensitivity at short  $\lambda$

» Light penetration into depletion region...

• Power absorbed in depletion region...

$$P(w) = P_i e^{-\alpha d} (1 - e^{-\alpha w}) (1 - R_f)$$

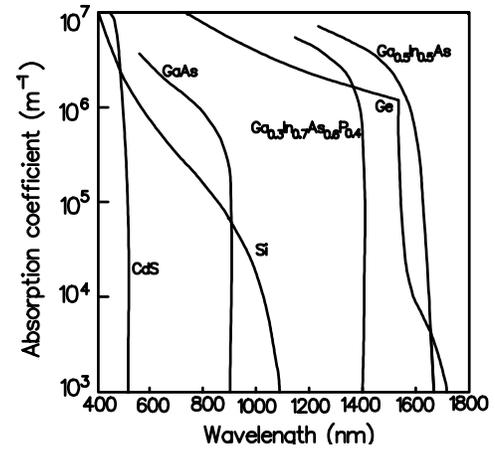
- Absorption coefficient:  $\alpha$
- Depletion region depth:  $w$
- Region begins at depth:  $d$
- Power reflectivity at detector surface:  $R_f$

• Increase  $w$  with  $i$  layer of  $pin$

• At short wavelengths

- $\alpha$  rises dramatically
- Strong surface absorption
- Little power penetrates

Fig. 6.3  
p. 201



## Detectors: Sensitivity

• Given by responsivity or quantum efficiency:

– **Responsivity**: output current per watt of optical power in

– **Quantum efficiency**: number of hole-electron pairs generated per photon

$$\mathcal{R} = I_{\text{out}} / P_{\text{in}}$$

$$\eta = Ihc / qP_i\lambda = hc\mathcal{R} / q\lambda$$

• Spectral response...

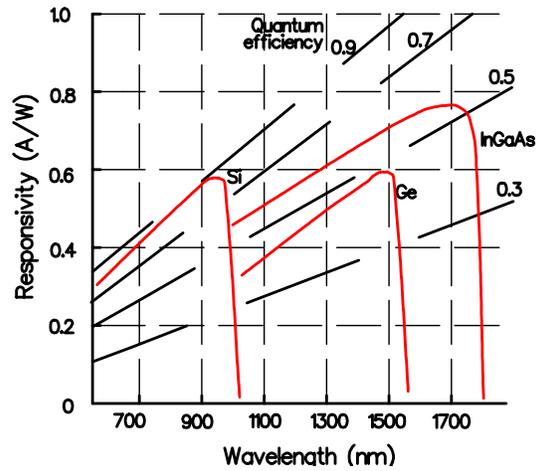


Fig. 6.5  
p. 203

## Detectors: Sensitivity - Probability Approach

- **Basic noise source: quantized nature of light**
- **Weak incident signal, output current not exact replica of ideal current**
  - Random generation of charge-carrier pairs by photons
    - » *Poisson random process*
      - Total carriers generated in time from  $t$  to  $t+T$  is random variable
      - Average number of carriers

$$\bar{N} = (\eta\lambda/hc) \int_t^{t+T} p(t) dt = (\eta\lambda/hc) E$$

–  $E$ : total energy in interval  $T$

- » Probability that number of charges created,  $N$ , equals specific number,  $n$ ,

$$P(N = n) = \bar{N}^n e^{-\eta\lambda E/hc} / n!$$

## Detectors: Probability Approach (cont.)

- Application of Poisson results...

- Want probability  $< 10^{-9}$  that “0” detected ( $N = 0$ ) when “1” transmitted; what  $E$  is needed?

$$P(N = n = 0) = \bar{N}^n e^{-\eta\lambda E/hc} / n! \leq 10^{-9}$$

$$P(N = 0) = \bar{N}^0 e^{-\eta\lambda E/hc} / 0! = 1 e^{-\eta\lambda E/hc} / 1 \leq 10^{-9}$$

$$E \geq 21hc/\eta\lambda \quad (\text{i.e., 21 photons})$$

- Require reception of 21 or more photons during bit period when a “1” is transmitted to ensure detection with error probability  $< 10^{-9}$

- If number of “1”s and “0”s equal and bit period is  $T_B$ ,

$$P_{\text{average}} \geq \frac{21hc}{2\eta\lambda T_B} = \frac{21hcB_R}{2\eta\lambda} \quad (\text{for } P_e \leq 10^{-9})$$

## Generalization of BER Results

- For any desired **BER (bit error rate)**, need...

$$e^{\left(-\frac{\eta\lambda E}{hc}\right)} \leq \text{BER} \quad \text{or} \quad E \geq \frac{hc}{\eta\lambda} \ln\left(\frac{1}{\text{BER}}\right)$$

- Minimum required average power is...

$$P_{\text{average}} = E_{\text{min}} / 2T_B = E_{\text{min}} B_R / 2$$

- Theoretical power required to achieve desired BER when limited by light quantization into photons of energy  $h\nu$

## Avalanche Photodiode: Physical Principles

- Differences from *pin* diodes...
  - Dope *p* and *n* regions higher
  - **Narrow *p* region added between *i* and *n*<sup>+</sup> region** (see notes below)
    - » Electric field in this region larger than in depletion region
      - Field accelerates carriers to high velocities
      - Collisions create more hole-electron pairs (***impact ionization***)
- Operating physics
  - Light enters through *p*<sup>+</sup> region and (ideally) absorbed in *i* region
  - Generated carriers separate and drift across *i* region
  - When electrons enter *p* region, accelerated and impact other atoms, creating more carriers
  - Carriers are accelerated and, in turn, create more carriers (***avalanche effect***)

## Avalanche Photodiode: Physical Principles(cont.)

- Avalanche multiplies photocurrent

- Multiplication factor:  $M = I_M/I$

- $I$ : output current without multiplication

- Instantaneous multiplication is random value;  $M$  is **average** multiplication

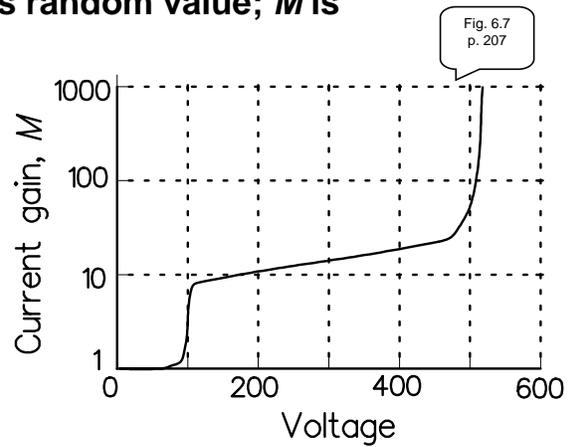
- Controlled by reverse bias

- »  $M=1$  expensive pin diode

- APD responsivity

$$\mathfrak{R}_{\text{APD}} = \eta q \lambda M / hc = M \mathfrak{R}_0$$

$\mathfrak{R}_0$ : responsivity at  $M = 1$



## Detectors: Signal-to-Noise Ratio

- Use **power signal-to-noise ratio**
  - “**Signal**” is signal **power** delivered to resistor by signal current
  - “**Noise**” is noise **power** delivered to same resistor
- **Signal-to-noise ratio, SNR**

$$\frac{S}{N} = \frac{P_{\text{signal}}}{P_{\text{noise}}} = \frac{\langle i_s^2 \rangle R}{\langle i_N^2 \rangle R} = \frac{\langle i_s^2 \rangle}{\langle i_N^2 \rangle}$$

- SNR independent of R; **need only mean-square signal current and mean-square noise current**
- Two noise mechanisms with photodiodes...
  - Shot noise and...
  - Thermal noise

## Detectors: Shot Noise

- Associated with quantization of charge or light
- Mean-square noise current:

$$\langle i_N^2 \rangle_{\text{shot noise}} = 2qIB \quad (I = I_L + I_{\text{dark}})$$

- $I$ : dc current of device
- $B$ : **electronic bandwidth**
- *pin* diode dc current:
  - dc output current due to incident light ( $I_L = \mathcal{R}P$ )
  - $I_{\text{dark}}$ : **dark current**
    - » dc current with no input illumination (e.g., thermal generation and surface leakage currents)
    - »  $I_{\text{dark}}$  in long- $\lambda$  detectors ~10x to 100x silicon short- $\lambda$  detectors
    - » In APDs...
      - **Amplified** bulk dark current,  $I_{\text{bulk}}$
      - **Unamplified** surface currents,  $I_{\text{surface}}$ 
        - Can be made zero with guard-ring design

## APDs: Excess Shot Noise

- Avalanche process contributes more noise described

$$\langle i_N^2 \rangle_{\text{shot APD}} = 2qI \Big|_{M=1} M^2 BF(M)$$

–  $F(M)$ : **excess noise factor**

» Extra noise added by avalanche process

» Depends on...

- Detector material
- Shape of  $E$  field
- Relative ionization rates

» Modeled as...

$$F(M) \approx kM + (1-k) \left( 1 + \frac{1}{M} \right) \approx M^x$$

Material	$k$	$x$
Silicon	0.02-0.04	0.3-0.5
Germanium	0.7-1.0	1.0
InGaAs	0.3-0.5	0.5-0.8

## Detectors: Thermal Noise

- Any resistive load (or device with associated resistance) produces noise
- Mean-square thermal noise current...

$$\langle i_N^2 \rangle_{\text{thermal}} = \frac{4kTB}{R}$$

***T***: noise temperature

***B***: electronic bandwidth

***R***: resistance value

– Assumes power delivered to matched load ( $R_L = R$ )

## Detectors: Signal-to-Noise Analysis

### •SNR of detector loaded by resistor $R_L$

$$\frac{S}{N} = \frac{\left\langle i_s^2 \right\rangle_{M=1} M^2}{2q(I_L|_{M=1} + I_{Dark\ bulk}) M^2 F(M) B + 2qI_{surf} B + (4kTB/R_L)}$$

– Numerator: mean-square signal current

– Denominator

» (Amplified) shot noise due to

• dc signal current ( $I_L$ ) (*before amplification*)

• Bulk dark current ( $I_D$ )

» Shot noise due to surface-leakage dark current  $I_{surf}$

» Thermal noise due to load resistor

## SNR of pin Diode

- For pin photodiode

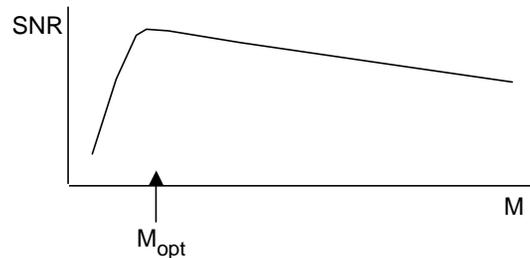
- $M = F(M) = 1$
- Dominant noise source usually thermal noise
- Constant input pulse...

$$i_s = \eta q \lambda P / hc = \mathfrak{R}P \quad \langle i_s^2 \rangle = \mathfrak{R}^2 P^2$$

$$\text{SNR}_{\text{pin diode}} \approx \frac{\mathfrak{R}^2 P^2}{4kTB/R_L}$$

## SNR of APDs

- For small  $M$ : thermal noise dominant; SNR increases with  $M$
- Large  $M$ :  $M^2F(M)$  makes shot noise dominant; SNR decreases with  $M$



–SNR has a maximum SNR at optimum  $M$

- Optimum  $M$ :

$$M_{\text{opt}} = \left( \frac{2qI_{\text{surf}} + (4kT/R_L)}{xq(I_L + I_D)} \right)^{\frac{1}{2+x}}$$

- Si APDs:  $M_{\text{opt}} = 80 \rightarrow 100$ 
  - SNR improves 40x  $\rightarrow$  50x (16  $\rightarrow$  17 dB)
  - Excessive noise in long- $\lambda$  APDs restricts use

## Analog Analysis

- **Modulated wave...**

$$p(t) = P(1 + m(t))$$

$$i(t) = M \Re_0 P(1 + m(t)) = I_L(1 + m(t))$$

$$I_{\text{DC}} = M \Re_0 P;$$

$$i_s(t) = M \Re_0 P m(t); \quad i_s(t)|_{M=1} = \Re_0 P m(t)$$

$$\langle i_s^2(t) \rangle|_{M=1} = \Re_0^2 P^2 \langle m^2(t) \rangle$$

- **Need to assume a specific  $m(t)$  to proceed...**

## Analog Analysis (cont.)

- Assume...

$$p(t) = P(1 + m \cos(\omega t))$$

$$i(t) = M \Re_0 P(1 + m \cos(\omega t)) = I_L(1 + m \cos(\omega t))$$

$$I_{\text{DC}} = M \Re_0 P;$$

$$i_s(t) = M \Re_0 P m \cos(\omega t); \quad i_s(t)|_{M=1} = \Re_0 P m \cos(\omega t)$$

$$\langle i_s^2(t) \rangle|_{M=1} = \Re_0^2 P^2 m^2 \underbrace{\langle \cos^2(\omega t) \rangle}_{1/2} = \frac{\Re_0^2 P^2 m^2}{2}$$

- Find SNR...

## Analog Analysis (cont.)

- SNR calculation...

$$\begin{aligned}
 \frac{S}{N} &= \frac{\langle i_s^2 \rangle \Big|_{M=1} M^2}{2q(I_L \Big|_{M=1} + I_D)M^2 F(M)B + 2qI_{\text{surf}}B + (4kTB/R_L)} \\
 &= \frac{\frac{\mathfrak{R}_0^2 P^2 m^2 M^2}{2}}{2q \left( \underbrace{I_L \Big|_{M=1}}_{\mathfrak{R}_0 P} \right) M^2 F(M)B + 2qI_D M^2 F(M)B + 2qI_{\text{surf}}B + (4kTB/R_L)} \\
 &= \frac{\frac{\mathfrak{R}_0^2 P^2 m^2 M^2}{2}}{2q(\mathfrak{R}_0 P)M^2 F(M)B + 2qI_D M^2 F(M)B + 2qI_{\text{surf}}B + (4kTB/R_L)}
 \end{aligned}$$

- Make some simplifying assumptions...

## Analog Analysis (cont.)

- Pin diode ( $M=1$ ) – assume that illumination is weak and, so, thermal noise is dominant...

$$\frac{S}{N} = \frac{\frac{\mathfrak{R}_0^2 P^2 m^2}{2}}{2q(\mathfrak{R}_0 P)B + 2qI_D B + 2qI_{\text{surf}} B + (4kTB/R_L)}$$
$$\approx \frac{\frac{\mathfrak{R}_0^2 P^2 m^2}{2}}{4kTB/R_L} = \frac{(1/2)\mathfrak{R}_0^2 P^2 m^2}{4kTB/R_L}$$

- Maximize SNR by...
  - maximizing  $m, \mathfrak{R}_0, P, R_L$
  - minimizing  $T, B$

## Analog Analysis (cont.)

- pin diode ( $M=1$ ) – assume that illumination is strong and, so, signal-dependent shot noise is dominant...

$$\frac{S}{N} = \frac{\frac{\mathfrak{R}_0^2 P^2 m^2}{2}}{2q(\mathfrak{R}_0 P)B + 2qI_D B + 2qI_{\text{surf}} B + (4kTB/R_L)}$$
$$\approx \frac{\frac{\mathfrak{R}_0^2 P^2 m^2}{2}}{2q(\mathfrak{R}_0 P)B} = \frac{\mathfrak{R}_0 P m^2}{4qB}$$

- “Quantum-limited” SNR
- Maximize SNR by...
  - maximizing  $m, \mathfrak{R}_0, P$
  - minimizing  $B$

## Analog Analysis (cont.)

- **APD...**

- Same SNR expression as general expression

$$\begin{aligned} \frac{S}{N} &= \frac{\langle i_s^2 \rangle \Big|_{M=1} M^2}{2q(I_L \Big|_{M=1} + I_D) M^2 F(M) B + 2qI_{\text{surf}} B + (4kTB/R_L)} \\ &= \frac{\mathfrak{R}_0^2 P^2 m^2 M^2}{2q(\mathfrak{R}_0 P) M^2 F(M) B + 2qI_D M^2 F(M) B + 2qI_{\text{surf}} B + (4kTB/R_L)} \end{aligned}$$

- **$M_{\text{opt}}$  exists; same value as before (Eq. 6.42b on p. 200)**

$$M_{\text{opt}} = \left( \frac{2qI_{\text{surf}} + (4kT/R_L)}{xq(\mathfrak{R}_0 P + I_{\text{Dark}})} \right)^{\frac{1}{2+x}}$$

- **$M_{\text{opt}} \rightarrow 1$  as power increases;**

- APDs have no advantage with strong illumination

## Noise-Equivalent Power

- **Noise-Equivalent Power (“NEP”)**

- Input power that makes the **SNR = 1**

- **Ex. Pin diode with weak signal**

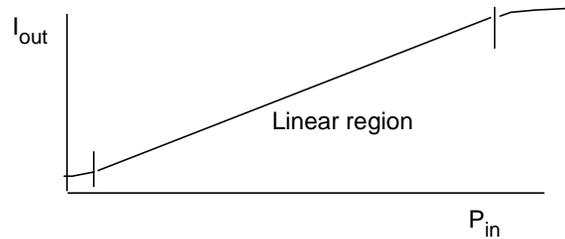
$$\begin{aligned}\frac{S}{N} &= \frac{\mathfrak{R}_0^2 P^2 m^2}{2q(\mathfrak{R}_0 P)B + 2qI_D B + 2qI_{\text{surf}} B + (4kTB/R_L)} \\ &\approx \frac{\mathfrak{R}_0^2 P^2 m^2}{2} = \frac{(1/2)\mathfrak{R}_0^2 P^2 m^2}{4kTB/R_L} \\ 1 &= \frac{(1/2)\mathfrak{R}_0^2 \text{NEP}^2 m^2}{4kTB/R_L} \Rightarrow \text{NEP} = \frac{\sqrt{8kTB/R_L}}{\mathfrak{R}_0 m}\end{aligned}$$

- **Other cases are similar**

- Find SNR; set SNR= 1 and let  $P = \text{NEP}$ ; solve for NEP

## Detectors: Linearity

- **Linearity**: of output current vs. optical input power curve



- Required for analog signal fidelity
- PIN diodes:
  - Excellent
  - Typically linear over 6 decades of input
- APDs:
  - Not quite as good
  - High linearity usually not required for weak signals

## PIN Diode: Speed of Response

- Factors

1. **Transit time**

- » Time to *drift* across depletion region:  $\tau = w/\langle v \rangle$

- $\langle v \rangle$ : Scattering-limited velocity (Si:  $1.0 \times 10^5$  m/s)
- Depletion width of  $10 \mu\text{m}$ ; response time  $\approx 0.1$  ns ( $\sim 10$  GHz bandwidth)
- Minimize by making  $w$  small (decreases sensitivity)

2. **Diffusion time**

- » Time for carriers created in  $p$  or  $n$  material (close to depletion region boundary) to diffuse into depletion region

- Diffusion process is *slow*
- Small fraction of carriers involved

- » Minimize by ensuring that most of carriers generated in depletion region

- Make  $w$  large ( $w \gg 1/\alpha$ )
  - [Increased depletion region increases transit time, however]

## PIN Diode: Speed of Response (cont.)

### 3. **RC time constant** of device and associated circuitry

– Bandwidth limitation:  $B_{max} = 1/2\pi RC_d$

»  $R$ : input resistance of preamplifier in parallel with load and device resistance (keep small for fast receiver,  $R \sim 50 \Omega$ )

»  $C_d$  **device capacitance**:  $= \epsilon A/w$

• Reduce  $C_d$  by making  $A$  small (decreased sensitivity) and  $w$  large (increases transit time, causing tradeoff)

– Usual compromise:  $w \approx 2/\alpha$

• Typical  $C_d < 1$  pF

• Primary limit in well-designed, fast pin diode (used in low-resistance circuit): transit time across depletion region

• Fast silicon devices have response  $< 1$  ns (multi-GHz bandwidths)

## APD: Speed of Response

- Response typically **slower than fast *pin* diodes**
  - Carriers must drift into avalanche region
  - Created carriers must drift back,
    - » Makes **total transit time ~2x longer**
- **Constant gain-bandwidth product** constraint
  - Caused by giving avalanche process time to occur
  - Typical value:  $M \cdot BW \leq 200$  GHz

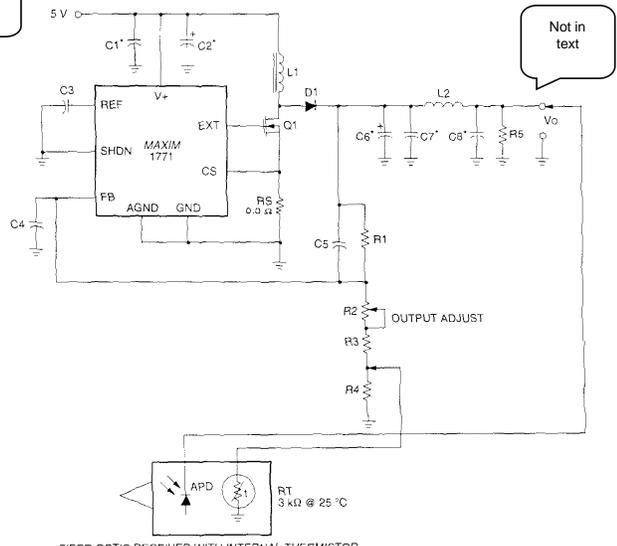
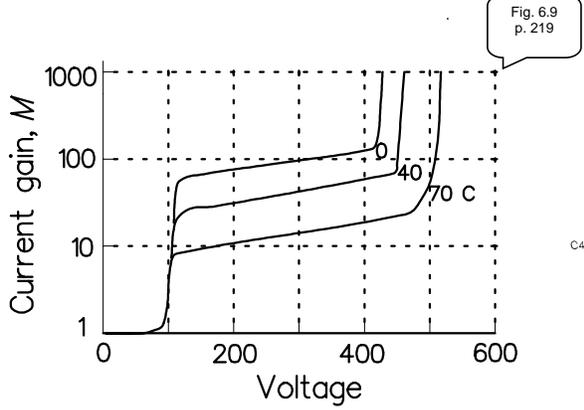
## Detectors: Reliability

- **No major problem**
- **Based on accelerated-temperature lifetime testing**
  - **Projected lifetime:  $\sim 10^8$  hours**

# APDs: Temperature Sensitivity

• *M* quite temperature sensitive

• Use temperature-compensating feedback circuit to minimize effect



FIBER-OPTIC RECEIVER WITH INTERNAL THERMISTOR  
 \* These capacitors must be low ESR types. 1-1002 (C)

Rcvrs-34

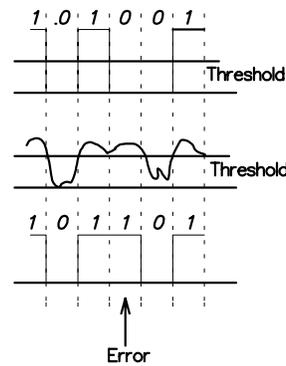
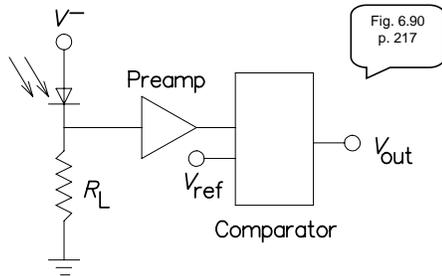
Table 1. Component List for Maxim dc-dc APD Bias Circuit

Designation	Description	Source/Part No.
Q1	Power MOSFET VBR(DSS) = 100 V, Id = 1.5 A	International Rectifier IRLL110
D1	Switching Diode VR = 120 V, IF = 400 mA	Central Semiconductor Corp. CMPD5001
L1*	Switching Inductor 330 μH Miniature Shielded, Surface Mount	Coilcraft Inc. DT1608C-334
L2	22 μH Surface-mount Inductor	Coilcraft Inc. 1812CS-223
C1, C7, C8	0.1 μF, 100 V Low ESR Ceramic	—
C2	330 μF, 10 V, Low ESR TA	Sprague Electric Company 594D337X0010R2T
C3	0.1 μF Ceramic	—
C4	1000 pF NPO	—
C5	22 pF NPO	—
C6	47 μF, 100 V Low ESR	—
R1	432 kΩ Surface Mount	—
R2	5 kΩ Single-turn Potentiometer	—
R3	9.53 kΩ Surface Mount	—
R4	2.74 kΩ Surface Mount	—
R5	20 kΩ Surface Mount	—
RS	0.2 Ω, 1/4 W, Low Inductance, Surface Mount	Dale Electronics WSL1206R00FRE4

\* Value not critical; 100 μH to 300 μH appears to work equally well.

## Detector Power and Bit-Error-Rate Revisited

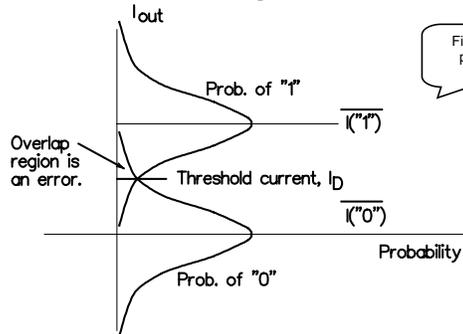
- **Digital receiver (left)**
  - Detects optical signal and converts to electrical signal
  - Decides whether electrical output represents “1” or “0” (using decision circuit), and...
  - Generates logic voltage output
- **Threshold voltage (decision level) critical to determining bit error rate (BER)**
  - Threshold expressed as fraction  $k$  of expected output of “1”
  - **Errors made due to noise (right)**



## Receivers: Noise Models

- Simplified noise assumptions...

- Detector output currents are Gaussian random variables



$$p(i_N) di_N = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(i_N - \bar{i})^2}{2\sigma^2}} di_N$$

- Mean value of current for logical "1":  $\overline{i(1)}$

- Mean value of output current for logical "0":  $\overline{i(0)}$  (= 0, later)

- Standard deviation  $\sigma$  is measure of width...

- » Mean square noise current:  $\langle i_N^2 \rangle = \sigma^2$

- » Assume standard deviations  $\sigma_1$  and  $\sigma_0$  are equal

## Receivers: Noise Models (cont.)

- Errors

- “0” sent: error if  $i_N$  positive and  $i_N > k\overline{i_N(1)} = I_D$

- “1” sent: error if  $i_N$  negative and  $|i_N| > (1-k)\overline{i_N(1)}$

- Total probability of error is...

$$P_e = P(0|1)P(1) + P(1|0)P(0)$$

- If  $P(1) = 1/2$  and  $P(0) = 1/2$

- » Combined error probability is...

$$P_e = (1/2)P\left[i_N < -(1-k)\overline{i_N(1)}\right] + (1/2)P\left[i_N > k\overline{i_N(1)}\right]$$

## Detectors: Threshold Location and BER

- Substituting Gaussian distribution, can show that probability of error is...

$$P_e = \text{BER} = \frac{1}{4} \left[ \text{erfc} \left( \frac{\bar{i}(1) - I_D}{\sigma_1 \sqrt{2}} \right) + \text{erfc} \left( \frac{I_D - \bar{i}(0)}{\sigma_0 \sqrt{2}} \right) \right] \quad (I_D \equiv k \bar{i}(1))$$

- Optimum threshold location to minimize BER is...

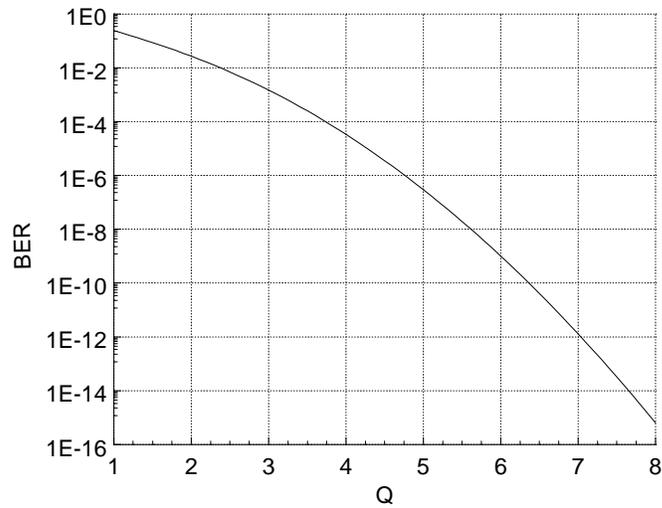
$$\frac{\bar{i}(1) - I_D}{\sigma_1} = \frac{I_D - \bar{i}(0)}{\sigma_0} = Q \Rightarrow I_D = \frac{\sigma_0 \bar{i}(1) + \sigma_1 \bar{i}(0)}{\sigma_0 + \sigma_1}$$

- BER with optimized threshold is...

$$\text{BER} = \frac{1}{2} \text{erfc} \left( \frac{Q}{\sqrt{2}} \right) \approx \frac{e^{-\frac{Q^2}{2}}}{Q\sqrt{2\pi}} \quad (\text{for } Q > 3)$$

## BER vs. Q

Fig. 6.13  
p. 223



- **Benchmarks:**

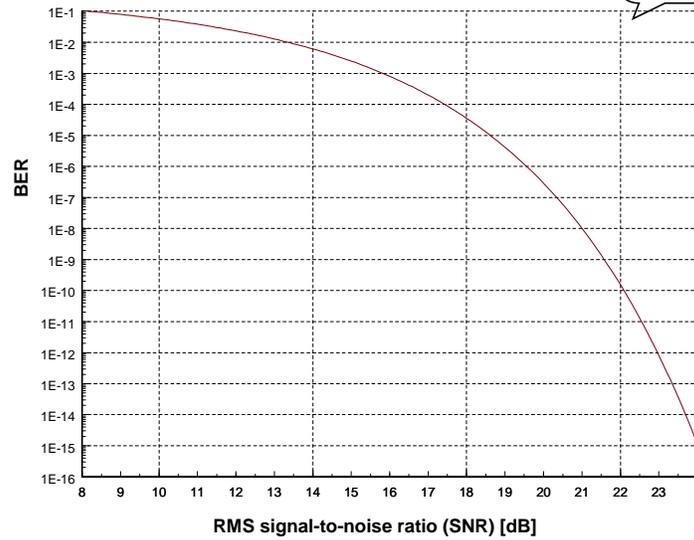
- Q = 4.76 (BER =  $10^{-6}$ )
- Q = 6.00 (BER =  $10^{-9}$ )
- Q = 7.04 (BER =  $10^{-12}$ )

## BER vs. SNR

- Assume

- $\sigma_0 = \sigma_1 = \sigma = \sqrt{\langle i_N^2 \rangle}$
- $I_D = [i(1) + i(0)]/2$  (i.e., midway)
- then...

$$\text{BER} = \frac{1}{2} \operatorname{erfc} \left( \frac{\sqrt{\text{SNR}}}{2\sqrt{2}} \right)$$



## Receivers: Minimum Required Power (cont.)

- **Sample example** (pp. 187–188, but BER =  $10^{-9}$ )
- **Begin with desired BER; find required SNR...**

» **BER =  $10^{-9}$  needs**  $\sqrt{\text{SNR}} = \sqrt{\langle i_s^2 \rangle} / \sqrt{\langle i_N^2 \rangle} = 21.5 \text{ dB} \Rightarrow 11.89$

**Limiting noise (e.g., thermal noise for pin diode or shot noise for APD), and calculate...**

$$\sqrt{\langle i_N^2 \rangle} = \sqrt{\frac{4kTB}{R_L}} = \sqrt{\frac{(4)(1.38 \times 10^{-23})(400)(10^7)}{50}} = 6.65 \times 10^{-8}$$

– **From required SNR, find  $\sqrt{\langle i_s^2 \rangle}$  ...**

$$\sqrt{\langle i_s^2 \rangle} = \sqrt{\text{SNR}} \sqrt{\langle i_N^2 \rangle} = (11.89)(6.65 \times 10^{-8}) = 7.90 \times 10^{-7} \text{ A}$$

– **Find the optical power required at the detector to achieve specified BER..**

$$P_{\min} = \sqrt{\langle i_s^2 \rangle} / \mathcal{R} = 7.90 \times 10^{-7} / 0.4 = 1.976 \times 10^{-6} \text{ W} = 1.976 \mu\text{W}$$

## Receivers: Noise and Sensitivity

- **Receiver front-end:**
  - Combination of detector and preamplifier
- Receiver noise properties set by...
  - » Detector
  - » Amplifier
    - Not detector alone
- Generally three common receiver implementations
  1. **Low-impedance front-end**
  2. **Integrating front-end**
  3. **Transimpedance amplifier front-end**
  - (Some do *not* fall into these categories)

## Receivers: 1. Low-impedance Front-End

- **Detector operates into low-impedance amplifier**
  - Usually  $50\ \Omega$  impedance
  - Ready availability of wideband RF amplifiers
- **Choose  $R_L$  equal to amplifier input resistance**
  - e.g.,  $50\ \Omega$  amplifier calls for  $50\ \Omega$  load
- **Poor preamplifier (not recommended)**
  - **Low sensitivity...**
    - Small voltage across amplifier & load resistance
    - High thermal noise from small load/amplifier resistance

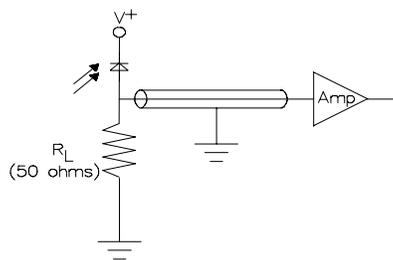
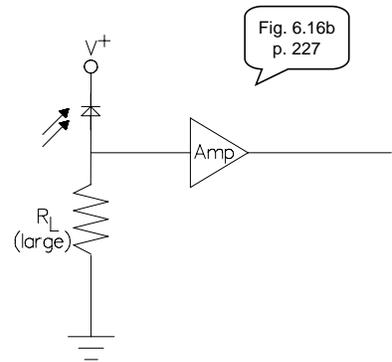


Fig. 6.16a  
p. 227

## Receivers: 2. High-impedance Front-End

- Amplifier has high  $R_{in}$ 
  - $R_L = R_{in}$
- Larger signal voltage and less thermal noise
  - Amplifier can use FET input for large  $R_{in}$
- Capacitances in parallel with load/amplifier resistance:
  - Total:  $C_T$ 
    - » Detector capacitance plus...
    - » Amplifier input capacitance plus...
    - » Parasitic capacitances
- Current generator driving parallel  $RC$  circuit
  - Integrator (*integrating front-end*)



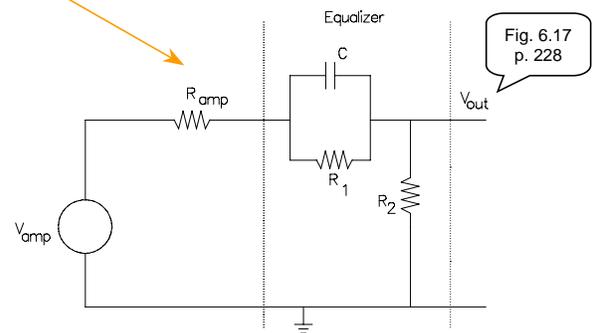
## Receivers: 2. High-impedance Front-End (cont.)

- **Bandwidth:**  $1/2\pi R_{parallel} C_T$ 
  - »  $R_{parallel}$ : 100s  $k\Omega \rightarrow$  few  $M\Omega$
  - »  $C_T$ : few pF or less
  - » Bandwidth  $\leq$  kHz range
    - Too low for high bit-rate
- **Equalization amplifier** compensates for low bandwidth (see example)
  - Choose  $R_1$  &  $C \Rightarrow 1/R_1 C = 1/R_L C_T$ 
    - » Equalizer zero cancels front-end pole
    - » Combined bandwidth  $>$  front-end bandwidth

$$\frac{V_{out}}{V_{amp}} = \frac{R_2(1 + j\omega R_1 C)}{R_1 + R_{amp} + R_2 + j\omega R_1 C(R_{amp} + R_2)}$$

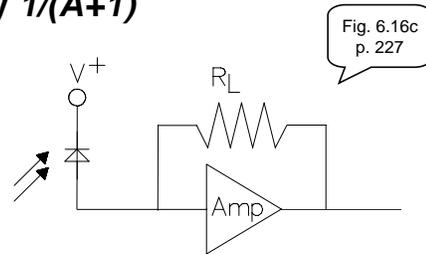
$$f_{combined} = \frac{1}{R_1 C} \frac{R_1 + R_{amp} + R_2}{R_{amp} + R_2}$$

- **Pro:** best sensitivity of all configurations
- **Con:**
  - Requires additional circuit
  - Limits dynamic range
  - Limits dc response
    - » Integrated low-frequency components saturate preamp



## Receivers: 3. Transimpedance Front-End

- **Current-to-voltage convertor (gain of  $R_L$ )**
  - Bandwidth:  $A \text{ (amplifier gain)}/2\pi R_L C_T$
  - No equalization amp
- **Low-frequency components reduced by  $1/(A+1)$** 
  - Reduces amplifier saturation
  - Increases dynamic range
- **Pros:**
  - **Simple** (no equalization amp)
  - **Good bandwidth**
  - **Good dynamic range**
- **Cons:**
  - **More noise** (less sensitivity) than integrating front-end



## Receivers: Amplifier Noise Effects

- Actual receivers: amplifier noise dominates detector noise
- How to account for amplifier noise?

– Amplifier **noise figure  $F_n$**

- » Describes noise added by amplifier (see notes)
- » Usually specified in dB; convert to numerical value for formulas
- » Good low-noise amplifier:  $F_n < 3$  dB; otherwise,  $\geq 6$  dB

– SNR:

$$\frac{S}{N} = \frac{G^2 \mathfrak{R}^2 P^2 M^2}{2q(\mathfrak{R}P + I_{\text{dark}})G^2 M^2 F(M)B + 2qI_{\text{surf}}BG^2 + (4kTBF_n G^2/R_L)}$$

» *pin* diode:  $M = F(M) = 1$

» APD: SNR maximum at  $M_{\text{opt}}$

$$M_{\text{opt}} = \left( \frac{2qI_{\text{surf}} + (4kTF_n/R_L)}{xq(\mathfrak{R}P + I_{\text{dark}})} \right)^{\frac{1}{2+x}}$$

Additional noise due to amplifier

## Receivers: FET Front-Ends

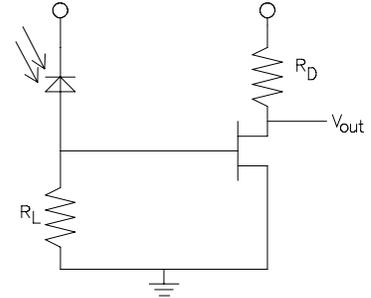
### 2. FET front-ends

- Can use either FETs or bipolar junction transistors

- FETs: superior noise properties
- GaAs microwave FETs for high-data-rate

- Representative common-source preamp

- Principal sources of noise
  - » Thermal noise from
    - FET channel resistance
    - Load resistor  $R_L$
  - » Shot noise due to FET gate leakage current
  - » Electronic  $1/f$  noise of FET
  - » See next page



## Receivers: FET Front-Ends (cont.)

### • Mean-square noise current of amplifier

$$\langle i_N^2 \rangle_{\text{amp}} = (4kT/R_L) \underbrace{I_2 B_R}_{\text{Bandwidth}} + 2qI_{\text{gate}} I_2 B_R + (4kT\Gamma/g_m)(2\pi C_T^2) f_c I_f B_R^2 + (4kT\Gamma/g_m)(2\pi C_T^2) I_3 B_R^3$$

Eq. 6.95  
p. 232

- $I_1, I_2, I_3,$  and  $I_f$ : *Personick integrals* (constants depending on input/output pulse shapes)
- $B_R$ : bit rate
- $R_L$ : load or feedback resistor
- $I_{\text{gate}}$ : FET gate leakage current
- $g_m$ : FET transconductance
- $C_T$ : total input capacitance
- $f_c$ : FET 1/f-noise corner frequency
- $\Gamma$ : FET channel noise factor
- Channel-noise factor  $\Gamma$  describes noise contribution from channel resistance and gate-induced noise
- $C_T = C_d + C_s + C_{gs} + C_{gd}$ 
  - »  $C_d$ : detector capacitance
  - »  $C_s$ : stray capacitance,
  - »  $C_{gs}$ : FET gate-to-source capacitance
  - »  $C_{gd}$ : gate-to-drain capacitance
- Corner frequency  $f_c$  of 1/f noise
  - » FET parameter
  - » Frequency where device 1/f electronic noise equals thermal noise of channel (characterized by  $\Gamma$ )
- Typical values shown in text and notes

## Receivers: FET Front-End Noise (cont.)

Eq. 6.95  
p. 232

$$\langle i_N^2 \rangle_{\text{amp}} = (4kT/R_L)I_2 B_R + 2qI_{\text{gate}}I_2 B_R + (4kT\Gamma/g_m)(2\pi C_T^2)f_c I_f B_R^2 + (4kT\Gamma/g_m)(2\pi C_T^2)I_3 B_R^3$$

- **First term**

- Thermal noise of load resistor
- Make resistor large...
  - » But reduces receiver dynamic range

- **Second term**

- Shot noise of gate leakage current
- Choose FET with low value of  $I_{\text{gate}}$

- **Third term**

- 1/f noise of preamp
- Choose FET with low 1/f noise (low value of  $f_c$ )

- **Fourth term**

- FET channel noise
- Choose FET with maximum value of  $g_m/C_T^2$

## Receivers: Noise in FET Front-Ends (cont.)

- High-bit-rate designs

- **Short-circuit common-source gain-bandwidth product**

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

- » Used to describe FET preamp wideband performance

- Usually interested in optimizing receiver performance at high bit rates...

- » Noise dominated by fourth term (due to  $B_R^3$  dependence) SO...

- » Minimum noise current:

$$\langle i_N^2 \rangle_{\text{amp min}} = (32kT) \frac{\Gamma(C_d + C_s)}{f_T} I_3 B_R^3 \quad (\text{for large } B_R)$$

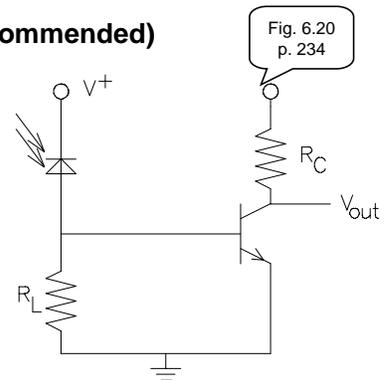
Eq. 6.98  
p. 233

- » Lowest amplifier noise when FET chosen that has maximum figure of merit of...

$$\text{FOM}_{\text{FET}} = \frac{f_T}{\Gamma(C_d + C_s)}$$

## Receivers: Noise in BJT Front-Ends

- Bipolar preamplifiers in some receiver front-ends
  - Low bit rates: noise higher than FETs (not recommended)
  - High bit rates: comparable noise
- Representative common-emitter preamp



- Principal noise sources:
  - Thermal noise from the load resistor  $R_L$
  - Shot noise due to base and collector bias currents ( $I_b$  and  $I_C$ )
  - Thermal noise from base-spreading resistance  $r_{bb'}$
- Amplifier mean-square noise current...

$$\langle i_N^2 \rangle_{\text{amp}} = (4kT/R_L)I_2B_R + 2qI_bI_2B_R + (2qI_c/g_m^2)(2\pi C_T^2)I_3B_R^3 + 4kTr_{bb'}[2\pi(C_d + C_s)]^2 I_3B_R^3$$

Eq. 6.100  
p. 234

## Receivers: Noise in BJT Front-Ends (cont.)

$$\langle i_N^2 \rangle_{\text{amp}} = (4kT/R_L)I_2B_R + 2qI_bI_2B_R + (2qI_c/g_m^2)(2\pi C_T^2)I_3B_R^3 + 4kTr_{bb'}[2\pi(C_d + C_s)]^2 I_3B_R^3$$

Eq. 6.100  
p. 234

- Parameters...

- $\beta$ : transistor current gain
- Transistor **transconductance** (depends on collector bias):

$$g_m = I_c/V_T \text{ (with } V_T = kT/q)$$

- Total capacitance:

$$C_T = C_d + C_s + C_{b'e} + C_{b'c}$$

$C_{b'e}$  and  $C_{b'c}$ : small-signal hybrid-pi model

- Capacitances depend on bias current (see notes or text)
- $f_T$ : "short-circuit common-emitter bandwidth product"

- There is **optimum bias current** to minimize noise

$$I_c \text{ optimum} = 2\pi C_0 f_T V_T \Psi(B_R)$$

Eq. 6.105  
p. 232

where  $\Psi(B_R) = 1/\sqrt{1 + (I_2 f_T^2 / \beta I_3 B_R^2)}$  and

$C_0$ : "total capacitance at zero bias"

$$C_0 = C_d + C_s + C_{b'c} + C_{je}$$

- Mean-square amplifier noise current at optimum bias...

See Eq. 6.110 on p. 233 or in notes below

- Minimize noise by maximizing...

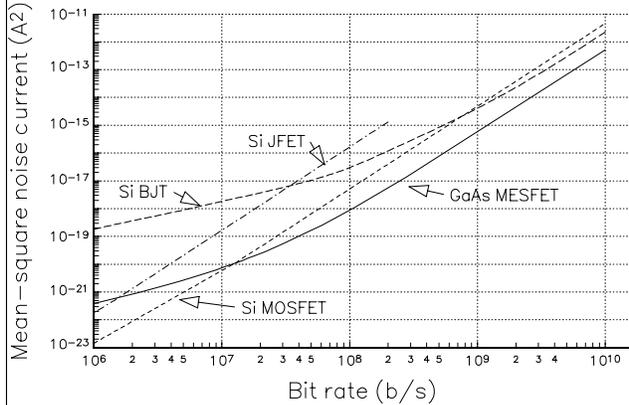
Eq. 6.111  
p. 235

$$\text{FOM}_{\text{BJT}} = \frac{2f_T}{C_0 + \pi f_T r_{bb'}(C_d + C_s)} \text{ (for large } B_R)$$

$$\approx \frac{2f_T}{C_0} \text{ (for large } B_R \text{ and small } r_{bb'})$$

## Comparison of Noise: FET and BJT Front-Ends

Fig. 6.21  
p. 236



- Parameter values in notes
- Low  $B_R$ : FET front-end superior to BJT
- High  $B_R$ : BJT comparable to FET
- FET front-ends...
  - Low  $B_R$ : Si MOSFET slightly advantageous
  - High  $B_R$ :
    - » GaAs MESFET slightly superior
    - » Si JFETs not suitable
- Relatively low gain-bandwidth product (lose gain above ~200 Mb/s)

## Receivers: Sensitivity of Detectors + Front-Ends

- **Power required on receiver**
  - To achieve BER in presence of *both...*
    - » **Detector noise *and...***
    - » **Amplifier noise**
- **First will consider pin receiver and, then, more-complicated case of APD receiver**

## Receivers: PIN Diode/Preamp Sensitivity

- >20 dB above quantum limit
  - Neglect signal-related shot noise
- Total mean-square noise current:  $\langle i_N^2 \rangle_{\text{Total}} = \langle i_N^2 \rangle_{\text{amp}} + 2qI_{\text{dark}}I_2B_R$
- Find required SNR for desired BER (from BER vs. SNR equation or curve)
- Detector power for pin diode receiver...

$$P = (hc\sqrt{\text{SNR}} / q\lambda) \sqrt{\langle i_N^2 \rangle_{\text{Total}}}$$

- $P$  calculated and plotted as function of  $B_R$ 
  - Once pin diode parameters and...
  - Amplifier type and parameters are known
- Straight-forward application of SNR concepts

## Receivers: APD/Preamp Sensitivity

- More difficult since...

- $M$  is additional variable and...
- Excess noise present
- $M_{opt}$  gives best sensitivity (depends on device, preamplifier noise, and  $B_R$ ; find by computer modeling or measurement )

- At  $M=M_{opt}$

- APD noise  $\approx$  preamplifier noise
- Dark-current shot noise:  $\langle i_N^2 \rangle_{\text{dark}} \approx 2qI_{\text{surface}} I_2 B_R + 2qI_{\text{D bulk}} M^2 F(M) I_2 B_R$
- Required power for an APD receiver is...

$$P \approx \left( \frac{hc}{q\lambda} \right) Q \left[ QqB_R I_1 F(M) + \sqrt{\frac{\langle i_N^2 \rangle_{\text{Total}}}{M^2} + 2qI_D F(M) B_R I_2} \right]$$

**Q:** Q-parameter required by BER

$I_1$  and  $I_2$ : Personick integrals

$$\langle i_N^2 \rangle_{\text{Total}} = \langle i_N^2 \rangle_{\text{amp}} + 2qI_{\text{surf}} I_2 B$$

- Continue on next slide...

## Receivers: APD/Preamp Sensitivity (cont.)

- If  $I_D$  is small enough that it adds negligible noise (true for short- $\lambda$  detectors)...

- Required receiver power simplifies to...

$$P = (hcQ/q\lambda) \left( \frac{\sqrt{\langle i_N^2 \rangle_{\text{Total}}}}{M} + qQB_R I_1 F(M) \right)$$

- Optimum gain

$$M_{\text{opt}} = (1/\sqrt{k}) \sqrt{\left( \sqrt{\langle i_N^2 \rangle_{\text{Total}}} / qI_1 B_R Q \right) - k + 1}$$

- If  $I_D$  not negligible (long- $\lambda$  detectors)...

- $M_{\text{opt}}$  smaller than value predicted ( $M_{\text{opt}}$  found graphically or numerically at each  $B_R$  by finding  $M$  that minimizes receiver power)
- Calculate total noise and sensitivity as function of  $B_R$

## Receivers: Extinction Ratio Effects

- **Extinction ratio:  $r = P(0)/P(1)$** 
  - Indicates if source turned off for “0”
- **Extinction ratio <1**
  - Reduces receiver sensitivity (*sensitivity “penalty”*)
    - » Shot noise associated with reception of “0”
    - » Not all of received optical power being modulated
  - **PIN diode receiver**
    - » **Power for desired BER is  $(1+r)/(1-r)$  larger**
  - **APD receiver**
    - » **Increases required power in complicated fashion**
    - »  **$r$  affects  $M_{opt}$  (found numerically)**

## Receivers: Eye Pattern Analysis

- Measures speed response and noise

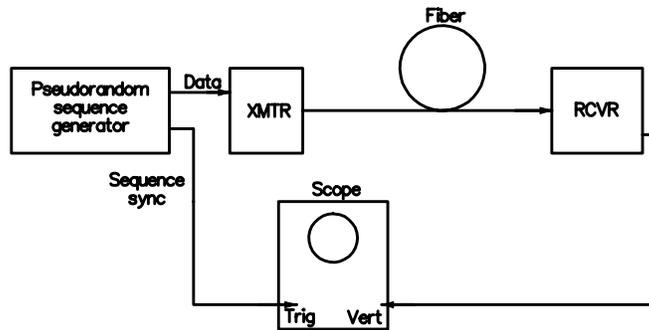


Fig. 6.22  
p. 240

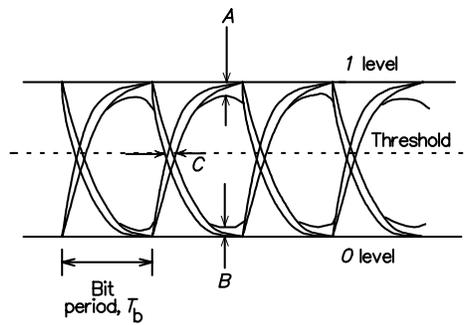
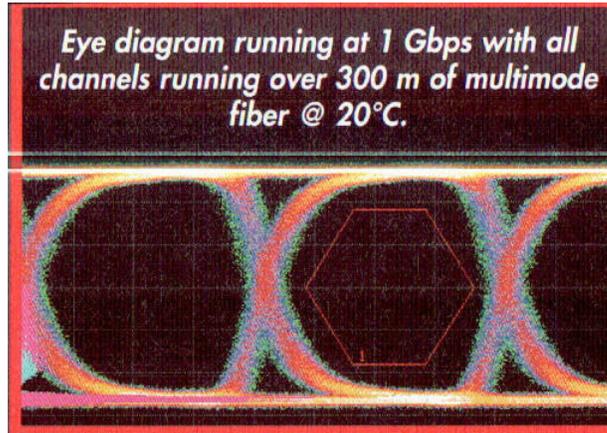


Fig. 6.23  
p. 240

## Example of Eye Pattern from Sampling 'Scope

- Experimental performance of high bit-rate link measured from eye *pattern*
- Time-domain measurement



- Pattern is superposition of outputs from pseudorandom stream of data pulses

## Spec Sheets

- See web site for examples of spec sheets
- [Receiver spec sheets](#)
- [Electronic receiver preamplifiers](#)

## Receivers: Summary

### • Properties of pin diodes and APDs

Tabel 6.8  
p. 241

	Photodiodes			APDs	
	Si	Ge	InGaAs	Si	Ge
$\lambda$ (nm)	400–1100	500–1800	1000–1500	400–1100	500–1650
Quant. Eff	80%	50%	70%	80%	75%
$t_{\text{rise}}$ (ns)	0.01	0.3	0.1	0.5	0.25
Bias (V)	15	6	10	170	40
$\mathfrak{R}_0$ (A/W)	0.5	0.7	0.4	0.7	0.6
M (gain)	1	1	1	80-150	80-150

- **Silicon detectors**
  - Mature technology
  - Operate close to theoretical limits in short- $\lambda$  region
- **InGaAs detectors**
  - Useful in long- $\lambda$  region
- **Germanium-based detectors**
  - Long- $\lambda$  detector
  - Fundamental difficulties with
    - » Noise performance
    - » Noise in APDs and
    - » High dark current

## Receivers: Summary (cont.)

- Noise contributions of preamplifier are important

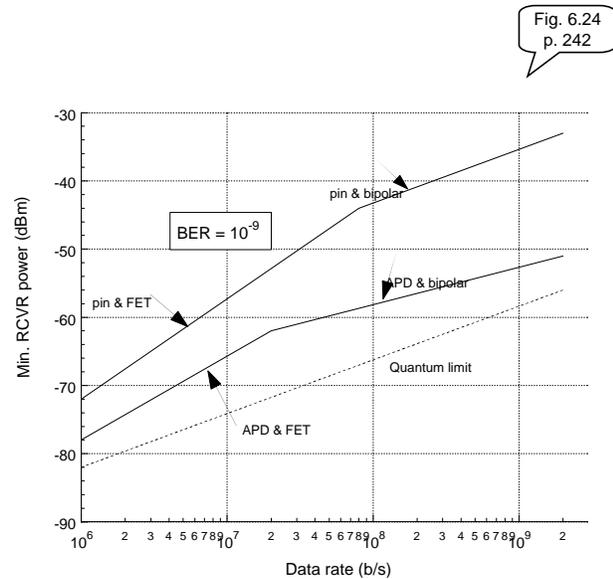
- High-impedance preamps

- » Pro: best sensitivity
- » Con: need equalization amplifier

- Transimpedance preamps

- » Pro:
  - Simple design/operation
  - Increased dynamic range
- » Con: increased noise
- » Frequently-used receiver

- Representative sensitivities for BER of  $10^{-9}$



## Receivers: Summary (cont.)

- **Observations...**
  - Increased sensitivity required at higher  $B_R$
  - Si FET receivers: good up to  $\sim 70 \text{ Mb}\cdot\text{s}^{-1}$
  - GaAs MESFETs: higher  $B_R$
- **APDs**
  - Pro:  $\sim 10 \text{ dB}$  increased sensitivity
  - Cons:
    - » More operating power required
    - » Higher cost
    - » Require temperature compensation
- **Best detector/preamplifier combination**
  - $\sim 10 \text{ dB}$  from quantum-limited detection