Image Distortion in Reconstructions from Phase-Only Holograms*

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Summary

METHERELL has proposed and demonstrated a method of acoustical holography which retains only the phase information in the acoustic waves scattered by an object [1]. Spatial amplitude variations of the object beam are disregarded. This type of hologram is of particular interest in computer holography since the amount of data used in the processing is reduced by half. Experiments performed by METHERELL and colleagues have shown that such a "phase-only" hologram can, in fact, produce a good reproduction of simple objects. Although there is little evidence of image distortion in METHERELL's experimental results, one might expect that some form of distortion would be introduced by the process.

This paper shows mathematically that for the particular case of FRAUNHOFER holograms, the image will exhibit a strong accentuation of intensity in regions having components of high spatial frequency (edges, etc.). For many purposes, this form of distortion is not detrimental because it results in little loss of resolution. Additionally, there can be a periodic nature to the reconstruction since the hologram exhibits a grating-like structure.

Using a digital computer we have also made a study of the reconstructed image of a back-irradiated slit where the system dimensions are comparable to those used in METHERELL's experiments. The theoretical images obtained in this way show a type of distortion which is similar, but less pronounced, than that found in the FRAUNHOFER phase-only holograms; that is, the intensity is greatly increased in the region of the edge.

Distorsion de l'image dans des reproductions d'après des hologrammes tenant compte seulement de la phase

Sommaire

METHERELL a proposé et démontré une méthode d'holographie acoustique qui retient seulement l'information de phase dans les ondes électriques dispersées par un objet [1]. On néglige les variations d'amplitude spatiale de faisceau. Ce type d'hologramme est d'un intérêt particulier dans l'holographie pour calculateur puisque la somme des données employées dans le procédé est réduite de moitié.

Les expériences présentées par METHERELL et ses collègues ont démontré qu'un tel hologramme «ne tenant compte que de la phase» peut en fait fournir une bonne reproduction d'objets simples. Bien qu'une distorsion de l'image soit peu évidente dans les résultats expérimentaux de METHERELL, on peut s'attendre à ce que quelque forme de distorsion se présente dans le procédé.

Cet article démontre mathématiquement que pour le cas particulier des hologrammes de FRAUNHOFER, l'image montrera une forte accentuation de l'intensité dans des régions ayant des composantes de haute fréquence (bords, etc. ...). Pour beaucoup de projets, cette forme de distorsion n'est pas préjudiciable parcequ'elle aboutit à une faible diminution de la résolution. En outre, il peut y avoir une nature périodique de la reproduction puisque l'hologramme présente une structure en forme de grille.

En utilisant un calculateur à main nous avons également fait une étude de l'image reproduite d'une fente éclairée par derrière, dans laquelle les dimensions du système sont comparables à celles utilisées dans les expériences de METHERELL. Les images théoriques obtenues de cette façon présentent un type de distorsion similaire à celle trouvée dans les hologrammes de FRAUNHOFER en ne tenant compte que de la phase, mais moins prononcée; c'est-à-dire que l'intensité est grandement accrue dans la région du bord.

Bildverzerrungen bei der Rekonstruktion von Phasen-Hologrammen

Zusammenfassung

METHERELL zeigte eine Methode für akustische Holographie, die nur die Phaseninformation der am Objekt gestreuten akustischen Welle enthält [1]. Räumliche Amplitudenschwankungen des gestreuten Strahls werden dabei nicht berücksichtigt. Diese Art von Hologrammen ist für die Computerholographie besonders interessant, da nur die halbe Menge von Daten erforderlich ist. In ihren Experimenten zeigten METHERELL und seine Mitarbeiter, daß ein solches Phasen-Hologramm in der Tat gute Reproduktionen von einfachen Objekten liefern kann. Obwohl es in METHERELLs experimentellen Ergebnissen

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nur eine leichte Bildverzerrung zu geben scheint, könnte man erwarten, daß bei dem Verfahren ganz bestimmte Verzerrungen auftreten könnten.

Die vorliegende Arbeit zeigt mathematisch, daß für den Spezialfall von FRAUNHOFER-Hologrammen das Bild eine Intensitätsanhebung in Gebieten mit hohen Raumfrequenzkomponenten, zum Beispiel Kanten, zeigt. Diese Art von Verzerrung ist für viele Zwecke nicht nachteilig, da sie nur in einem geringen Verlust an Auflösung resultiert.

Mit einem Digitalrechner wurde außerdem das rekonstruierte Bild eines rückseitig beleuchteten Spalts untersucht, wobei die Dimensionen mit denen in METHERELLs Experimenten vergleichbar waren. Die auf diese Weise erhaltenen Bilder zeigen eine Verzerrung, die ähnlich ist, aber stärker als erwartet, wie die, die bei FRAUNHOFERschen Phasen-Hologrammen gefunden wurde. Die Intensität steigt im Bereich von Kanten stark an.

1. Introduction

METHERELL has proposed and demonstrated a modified method of acoustic holography [1] which uses only half the data present in the acoustic wave scattered by the object. Such a feature can be advantageous in computer holography by reducing the storage requirements as well as the processing time. In this method he uses a scanning microphone and phase-lock circuit to detect only the phase of the complex wave distribution scattered by an object. He adds in the reference beam electronically and displays the intensity of the sum on a cathode-ray oscilloscope in a raster form. A photograph of the tube face provides the hologram. Reconstructions of the "phase-only" hologram can, in fact, produce a good image with little apparent deterioration or loss of resolution. It can be argued that this technique has an advantage over the conventional method in the sense that the most desirable holographic system is one which attains the highest quality of reconstructed image from the least recorded data. While there is little visible evidence of it in METHERELL's results, we might intuitively expect some loss of information to result from omitting the amplitude variations of the object beam.

2. Theory

Consider first a single slit in the analytically simple FRAUNHOFER region. Assume that the slit is of width $2a$ and is placed in a simple holographic system as shown in Fig. 1. A phase-only hologram of the slit could be made with this scheme by providing a means of eliminating the amplitude variation in the object beam before it reaches the hologram plane. We will examine mathematically the conventional hologram and its reconstructed real image and then do the same for the phase-only hologram. Inspection of the images will illustrate the differences of the two types of holograms.

If the distance of the recording medium from the object is great enough so that the diffraction pattern is in the FRAUNHOFER region, the recorded hologram is then of the sideband FRAUNHOFER

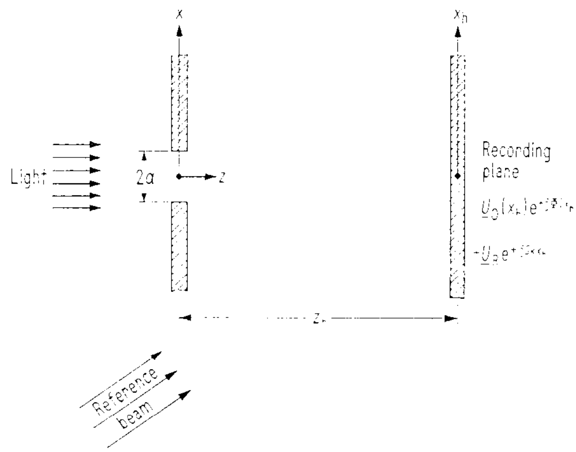

Fig. 1. General recording arrangement for holograms.

type. Under these circumstances the complex amplitude distribution for the object beam at the holographic plane is given by

$$\underline{U}_{0h}(x_h, z_h)\, e^{-j\omega t} = \underline{U}_0(x_h)\, e^{-j\varphi(x_h)}\, e^{-j\omega t} = \left\{ \frac{e^{jkz_h}}{[j\lambda z_h]^{1/2}} \left(F\{\text{object}\}\right)_{f_x = \frac{x_h}{\lambda z_h}} \right\} e^{-j\frac{k}{2z_h}x_h^2}\, e^{-j\omega t} \tag{1}$$

where

$$\left(F\{\text{object}\}\right)_{f_x = \frac{x_h}{\lambda z_h}}$$

is the spatial FOURIER transform of the object with the spatial frequency evaluated at $\dfrac{x_h}{\lambda z_h}$ and x_h is the distance in the x direction measured at the hologram plane.

For the single slit the complex amplitude distribution is

$$\underline{U}_0(x_h)\, e^{\mp j\varphi(x_h)} = \left\{ \frac{e^{jkz_h}}{[j\lambda z_h]^{1/2}}\, 2a\, \frac{\sin\left(2\pi \frac{x_h}{\lambda z_h}\right)a}{2\pi \frac{x_h}{\lambda z_h}\, a} \right\} e^{\mp j\frac{k}{2z_h}x_h^2} = \tag{2}$$

$$= \left\{ \frac{e^{jkz_h}}{[j\lambda z_h]^{1/2}}\, 2a\, \frac{\sin\left(2\pi \frac{x_h}{\lambda z_h}\right)a}{2\pi \frac{x_h}{\lambda z_h}\, a} \right\} e^{\mp j\left(\left(\frac{k}{2z_h}\right)x_h^2 - f_x x_h\right)}$$

where $f(x_h)$ has the value $0(\pm 2n\pi,\ n$ an integer) whenever the function,

$$\frac{\sin 2\pi \dfrac{x_h}{\lambda z_h}}{2\pi \dfrac{x_h}{\lambda z_h}},$$

is positive and the value $\pi(\pm 2n\pi)$ whenever that function is negative.

From this expression it is now easy to identify the amplitude function $\underline{U}_0(x_h)$ and the phase function $\varphi(x_h)$; the factor

$$\left\{ \frac{e^{jkz_h}}{[j\lambda z_h]^{1/2}}\, 2a\, \frac{\sin 2\pi \dfrac{x_h}{\lambda z_h} a}{2\pi \dfrac{x_h}{\lambda z_h} a} \right\}$$

being the amplitude function, and $\frac{k}{2z_h} x_h^2 + f(x_h)$, the phase function. In the phase-only process the phase information alone is retained; the amplitude function is replaced by a constant.

For a general hologram the complex wave distribution is added to an off-axis wave represented by $\underline{U}_r e^{+jSkx_h}$ to form the hologram. Here $\underline{U}_r$ is a complex constant of position and S is a shorthand notation for the sine of the angle which the reference beam makes with the z axis. The intensity pattern due to the above light is recorded on photographic film and a positive transparency is made. The transmission function of the transparency is proportional to the intensity pattern; hence we have

$$\begin{aligned} t = KI = K\frac{UU^*}{2} = \frac{K}{2}\,[\underline{U}_r \underline{U}_r^* + \underline{U}_0(x_h)\,\underline{U}_0^*(x_h) + \\ + \underline{U}_r \underline{U}_0^*(x_h)\, e^{+j(Skx_h - \varphi(x_h))} + \\ + \underline{U}_r^* \underline{U}_0(x_h)\, e^{-j(Skx_h - \varphi(x_h))}]. \end{aligned} \tag{3}$$

The reconstruction is performed by illuminating the developed hologram with a planar beam of laser light which is antiparallel to the original reference beam (i.e., by the conjugate of the reference beam). This technique will give a real image which is on-axis and located at the precise position of the actual object. The beam which then emerges from the hologram is given as follows:

$$\begin{aligned} \underline{U}e(x_h, z_h) &= t\,\underline{U}_r^* e^{-jSkx_h} = \\ &= \frac{K}{2}[\underline{U}_r \underline{U}_r^* \underline{U}_r^* e^{-jSkx_h} + \\ &+ \underline{U}_0(x_h)\,\underline{U}_0^*(x_h)\underline{U}_r^* e^{-jSkx_h} + \\ &+ \underline{U}_r \underline{U}_r^* \underline{U}_0^*(x_h)\, e^{-j\varphi(x_h)} + \\ &+ \underline{U}_r^* \underline{U}_r^* \underline{U}_0(x_h)\, e^{-j(2Skx_h - \varphi(x_h))}]. \end{aligned} \tag{4}$$

Holographic theory [2] provides the interpretation of these terms. The first term corresponds to a replica of the reference beam making an angle Θ with the axis. The second term represents noise. Because of its dependence on the amplitude function of the scattered wave, we call it the "amplitude noise" term. The third term is the one that gives the real image. As described above, this term is on-axis. The fourth term represents the virtual image. It is found at an angle 2Θ off the axis.

Focusing attention on the third term, we realize that if we go a distance $-z_h$ along the z axis from the hologram, we should reproduce a real image of the original object at the origin. Since z_h is large enough to be in the FRAUNHOFER region, we can find the complex amplitude distribution function at the image plane by using the FRAUNHOFER diffraction equation again. Thus for the slit

$$\underline{U}_i(x, 0) = \left[\frac{K\underline{U}_r\underline{U}_r^*}{2}\right] e^{+j\frac{k}{2z_h}x^2} \times \\ \times \int_{-\infty}^{\infty} \frac{\sin 2\pi \dfrac{x_h}{\lambda z_h} a}{\pi x_h}\, e^{-j\frac{kx_h^2}{2z_h} - j2\pi\left(\frac{x_h}{\lambda z_h}\right)x}\, dx_h. \tag{5}$$

By invoking the FRAUNHOFER approximation (i.e., the same approximation which allowed the use of the above integral):

$$\frac{k}{2z_h} x_h^2 \ll 1, \quad \frac{k}{2z_h} x^2 \ll 1$$

we then obtain

$$\underline{U}_i(x, 0) \approx \frac{K\underline{U}_r\underline{U}_r^*}{2} \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases} \tag{6}$$

where the approximation sign indicates the application of the FRAUNHOFER approximation.

This expression for the conventional hologram reconstruction is precisely what we would expect to get by inspecting the third term of eq. (4). Since that term is a constant times the conjugate of the original object beam, it represents a family of rays having exactly the same ray paths as the original object beam but with the rays following those paths in the opposite direction. Thus from eq. (4) we expect a reconstruction of the original object beam at the object plane.

Hence the conventional system theoretically reconstructs the original object with no distortion from an infinite sideband FRAUNHOFER hologram.

Examining now the case of the phase-only hologram, we proceed as before, except that we use only the phase information in the original object beam and not the amplitude information. We thus replace $\underline{U}_0(x_h)$ in the original expression with a constant $\underline{U}_0$. Constructing the real image we have

$$\underline{U}_i(x, 0) = \frac{e^{jkz_h} e^{+j\frac{k}{2z_h}x^2}}{[j\lambda z_h]^{1/2}} \frac{K\underline{U}_r\underline{U}_r^*}{2} \times \\ \times \int_{-\infty}^{\infty} \underline{U}_0^* e^{-jf(x_h)} e^{-j\frac{k}{2z_h}x_h^2} e^{-j2\pi\frac{x_h}{\lambda z_h}x}\, dx_h. \tag{7}$$

The function $e^{-j\varphi(x_h)}$ (corresponding to the real image as it just emerges from the hologram plane) is shown in Fig. 2. Invoking the FRAUNHOFER

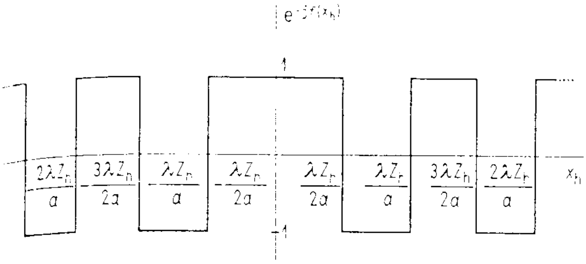

Fig. 2. Graphical representation of the emerging real image term at the hologram plane.

approximation and performing the FOURIER transform, we get

$$\underline{U}_i(x,0) \approx \frac{e^{+jkz_h}}{[j\lambda z_h]^{1/2}} \, \frac{2a}{\lambda z_h} \, \frac{K \underline{U}_r \underline{U}_r^*}{2} \, \underline{U}_0^* \, \frac{\tan\frac{\pi x}{2a}}{\frac{\pi x}{2a}} . \tag{8}$$

The image intensity distribution found by taking the magnitude squared of this function is shown in Fig. 3. The slit image reconstruction is an infinite set of intense lines running in the slit direction and located a distance $2a$ apart (i.e., the width of the

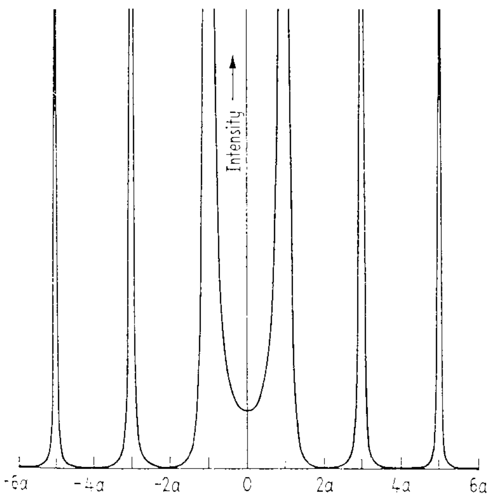

Fig. 3. Reconstructed image intensity distribution from a FRAUNHOFER phase-only hologram.

slit). This repetition of lines is the result of the grating-like structure of the hologram. Since the structure is not strictly periodic, the center of the reconstruction is uniquely located. Its location is determined by the presence of a non-zero intensity midway between two adjacent lines of the above infinite set. These two central lines can be regarded as defining the slit edges in the image. The widths of the other lines decrease with increasing distance away from the central axis and midway between these lines the intensity goes to zero. Hence for the FRAUNHOFER region, the reconstructed image of the slit appears with accentuated edges and with an infinite number of periodically distributed lines of light located on both sides of the slit image.

3. Analysis of a special case

In an effort to more closely duplicate the experiments performed by METHERELL and colleagues, we have analyzed a case with dimensions comparable to those in their experiment. The object was an infinite slit with a width (i.e., the value of $2a$) of 0.305 m with a 2 m wide recording plane located 2 m from the object. The wavelength of the illumination was 0.0186 m (sound in air at 18 kHz). For dimensions of this proportion the hologram is of the "very near-field" type, i.e., the distance to the recording plane is smaller than that for the FRESNEL region. This solution required a digital approach since the FRESNEL-KIRCHHOFF diffraction equation necessary for this problem does not lend itself easily to analytic solution. The analysis itself was limited to one dimension for computational simplicity. The holograms were also assumed to be of the GABOR type: that is, the reference beam was on-axis. This requires some minor changes in the mathematical representation of the hologram and the representation of the reconstruction. The hologram representation becomes

$$\begin{aligned} t = K\frac{\underline{U}\,\underline{U}^*}{2} = \frac{K}{2}\,[\,&\underline{U}_r \underline{U}_r^* + \underline{U}_0(x_h)\,\underline{U}_0^*(x_h) + \\ &+ \underline{U}_r \underline{U}_0^*(x_h)\,e^{-j\varphi(x_h)} + \\ &+ \underline{U}_r^* \underline{U}_0(x_h)\,e^{+j\varphi(x_h)}]. \end{aligned} \tag{9}$$

(Again it is noted for the phase-only hologram that the value of $\underline{U}_0(x_h)$ will be represented by a complex constant $\underline{U}_0$.) The reconstruction beam representation is

$$\begin{aligned} \underline{U}_e(x_h, z_h) &= t\,\underline{U}_r^* = \\ &= \frac{K}{2}\,[\,\underline{U}_r \underline{U}_r^* \underline{U}_r^* + \underline{U}_0^*(x_h)\,\underline{U}_0(x_h)\,\underline{U}_r^* + \\ &+ \underline{U}_r \underline{U}_r^* \underline{U}_0^*(x_h)\,e^{-j\varphi(x_h)} + \\ &+ \underline{U}_r^* \underline{U}_r^* \underline{U}_0(x_h)\,e^{+j\varphi(x_h)}]. \end{aligned} \tag{10}$$

Just as was the case with eq. (4), in this expression the first term is a replica of the reference beam, the second represents the noise which we call "amplitude noise", the third is the real image of the object that will be investigated and the fourth is the virtual image of the object.

For a GABOR hologram we note that all terms represent on-axis beam components. This implies that the resulting images will not be spatially separated automatically as in the side-band type of hologram, but that some special means must be used to separate the images. It is possible to use spatial filtering to eliminate the reference beam replica and the virtual image. For computational purposes, therefore, we eliminated these terms mathematically by representing the emerging beam at the hologram plane as

$$U_e'(x_h, z_h) = U_0(x_h)\, U_0^*(x_h) + U_0^*(x_h)\, e^{-j\varphi(x_h)} \tag{11}$$

where the reference beam U_r^* has been assumed to be unity. Here the emerging beam has terms representing the "amplitude noise" and the real image only. The assumption of a unit reference beam is arbitrary. With the reference of a greater magnitude the effect of the "amplitude noise" term compared with the real image term can be reduced.

Since our interest was only in the resulting image, eq. (11) was the logical starting point for the computer calculations. As written, it applies to the conventional hologram but we could easily modify eq. (11) to compute the image for the phase-only type also. The specific modifications for the calculations made will be discussed below.

To use eq. (11), it was necessary first to calculate $U_0(x_h)\, e^{+j\varphi(x_h)}$ by applying the FRESNEL-KIRCHHOFF integral to the propagation of the scattered wave from the slit to the hologram plane. To calculate the image, the emerging beam representation (that is, $U_e'(x_h, z_h)$) was then also put into the FRESNEL-KIRCHHOFF integral and the complex wave function computed for propagation to the image plane (a distance equal to that between the object and the hologram plane). The intensity of the image was then calculated and displayed graphically.

The computation procedure was tested by computing an image from the conventional hologram without the presence of any interfering noise. This implies dropping the "amplitude noise" term of eq. (11) and using only the real-image term which is simply the complex conjugate of the previously calculated object wave function $U_0(x_h)\, e^{+j\varphi(x_h)}$. The image was obtained as described in the preceding paragraph and is shown in Fig. 4. The reconstruction of the slit obviously is quite accurate in the positioning of the edges and in the representation of the intensity across the slit. The slit is not completely squared off because of the finite size of the recording plane which gives rise to the ripple shown.

After completing the above test, we investigated the phase-only case. For this case the modified emerging wave is simply

$$U_e'(x_h, z_h) = 1 + 1\, e^{-j\varphi(x_h)}\,. \tag{12}$$

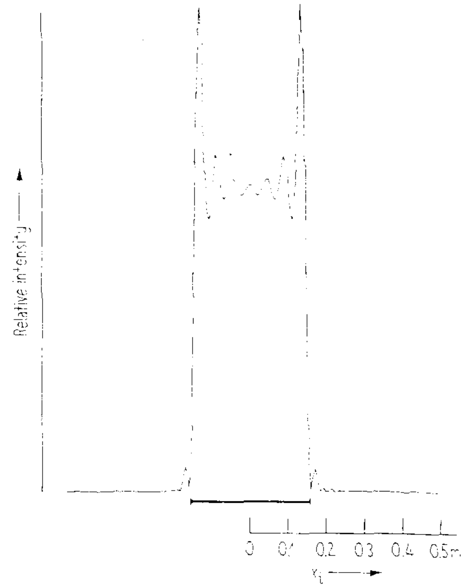

Fig. 4. Reconstructed image intensity distribution from a FRESNEL phase-amplitude "noiseless" hologram.

It is noted that the "amplitude noise" term is now constant and can be disregarded, as it, too, would be removed by the spatial filter. This implies that there will be no competing noise in the phase-only case. Taking the complex conjugate of the scattered wave function, dividing by its magnitude, and putting that into the FRESNEL-KIRCHHOFF integral gives the intensity pattern of Fig. 5. Note that the

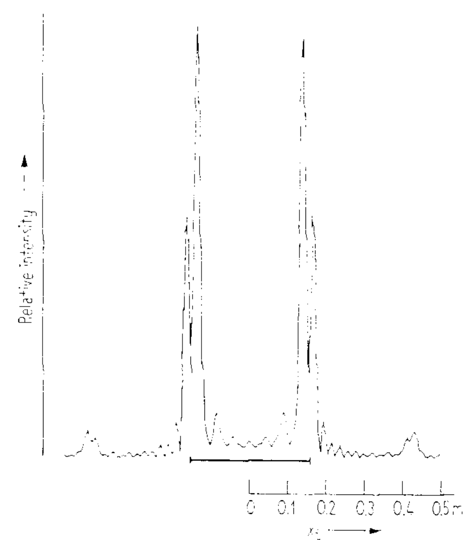

Fig. 5. Reconstructed image intensity distribution from a FRESNEL phase-only hologram.

positions of the edges of the slit are slightly misplaced and the intensity across the slit is misrepresented. The edges of the slit are accentuated by the "piling up" of light at their positions. This confirms a distortion seen in photographs of reconstructed images made by METHERELL and SPINAK [3] which show the edges of a board as being brightly outlined.

For the conventional hologram, the first term of eq. (11), the "amplitude noise" term, cannot be removed by spatial filtering since it varies across the recording plane. Since this term is therefore always included in the real image, it was instructive for purposes of comparison to consider it in the computed reconstruction. The "amplitude noise" term and the real image term were added as in eq. (11) and put into the diffraction integral with the results shown in Fig. 6. Here also the slit shape

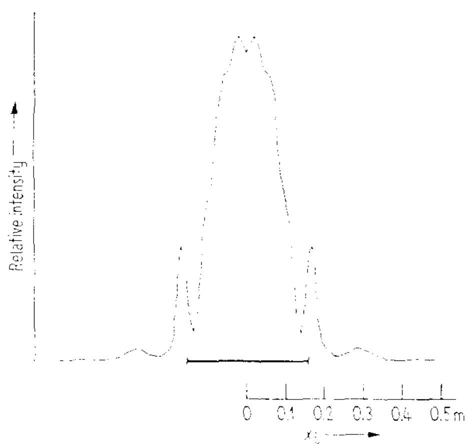

Fig. 6. Reconstructed image intensity distribution from a FRESNEL phase-amplitude hologram with "amplitude noise" added.

is distorted with high intensity at the center rather than an even spread of intensity across the slit. There is also competing light outside the region of the slit further degrading the image. The image resembles a slit but the edge positions are not well defined.

In the very near-field it is apparent that there are disadvantages and advantages to each type of hologram. It is obvious that the ideal reconstruction is produced by the conventional hologram without the presence of noise. This type gives a smooth distribution of intensity (except for the expected ripple) with minimal light outside the boundary of the slit. The edges themselves are properly located. In the presence of noise the reconstruction of the conventional hologram suffers in quality. The edges are ill-defined and the intensity falls off toward the edges. There is spurious light outside the boundary of the slit further detracting from the image. The reconstruction then represents a slit of poorly resolved edges with some extraneous background light. The phase-only reconstruction distorts the image by "piling up" the light at the edges, thereby accentuating them. The edges, although well defined, are somewhat misplaced, resulting in a slightly contracted image. In the phase-only case, as noted above, there is no "amplitude noise" which interferes with the image. This is obviously an advantage. The noise can be strong enough in many cases to be a serious detriment to the image. It would seem, therefore, that the phase-only hologram might be useful where interest is mainly in the shape or outline of the object or when noise is sufficiently strong. Under these circumstances, we can obtain a good representation of the object, although not of the exact dimensions.

4. Conclusion

In summary, this study has been an investigation of the quality of reconstruction of a phase-only hologram of a simple object — a long slit. It is apparent that real savings in the collection and processing of acoustic holographic data may be made possible by detecting only the phase of the scattered wave from the object. If the holographic distances are great enough to be in the FRAUNHOFER region, analytic methods show the phase-only hologram to be of limited use. In this region distortion of the intensity distribution is present. The intensity rises from the center to a maximum at the edges instead of remaining constant over the entire slit width. Additional lines of intense light are also present outside of the central image. If the hologram is in the very near-field, the phase-only hologram can be of significant value, mainly where the shape or outline of the object is of importance rather than its size or its intensity distribution. Also phase-only holograms do not have any competing "amplitude noise" when used in conjunction with the spatial filtering reconstruction setup. For some purposes such as outline recognition these properties of the very near-field, phase-only hologram might be advantageous.

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